Risk Sharing and Endogenous Group Formation.

Fernando JARAMILLO[*], Hubert KEMPF[†]and Fabien MOIZEAU[‡]

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[*] Universidad de Rosario, Bogota (Colombia), email: fjaramil@uniandes.edu.co.

[†] Banque de France and Paris School of Economics, email: hubert.kempf@univ-paris1.fr.

[‡] Toulouse School of Economics (GREMAQ), email: fabien.moizeau@univ-tlse1.fr.

1 Introduction.

- The basic issue: why are there limits to insurance? Why risk-bearing arrangements do not encompass the whole society?

- A basic issue that follows empirical investigations on perfect insurance (Townsend, 1994, 1995; see references in footnote 4 of Kazianga and Udry 2005). It has been found that households are able to protect consumption against adverse income shocks but full insurance is not achieved.

- The standard explanation (Kocherlakota, 1996, Ligon, Thomas and Worrall, 2002): due to "defection effects". Risk sharing arrangements as self enforcing contracts. Note: these contributions only consider individuals deviations.

- We provide an alternative explanation: group formation as an insurance scheme. Hence the segmentation of society plays a crucial role in providing differentiated insurance schemes to agents differentiately affected by risk. The limits of communities themselves cannot be considered exogenous to the management of risk. They are part of the answer!

- The relationship between risk and group formation: already studied. Genicot and Ray (2003) develop a group formation approach (also Taub and Chade (2002) and Dubois (2006)). We follow this approach.

What we do here:

- We study the complete segmentation of society into "risk-insuring groups". Characterization.

- We assume that agents precommit (as in Townsend). That is, individuals cannot deviate and leave their coalition after the realization of shocks.

- We study the impact of the variance schedule on segmentation and the management of risk when individuals are free to form coalitions.

- We characterize and discuss the optimal partition of society into "risk-managing coalitions".

- We discuss the empirical implications of this partitioning of society.

What we find:

- More (aggregate) risk do not lead to more segmentation.

- Explanation of empirical results.

2 The Model.

2.1 The society.

- We consider a population **I** comprised of $i = 1, ..., N$ individuals. Each individual endowment y_i has a deterministic component w_i and is affected by an idiosyncratic risk ε_i; $y_i = w_i + \varepsilon_i$. ε_i is normally distributed with variance σ_i^2: $\varepsilon \rightsquigarrow N(0, \sigma_i^2)$. Without loss of generality, we index individuals as follows: for i and $i' = 1, .., N$ with $i < i'$ then $\sigma_i^2 < \sigma_{i'}^2$. We will thus say that a lower indexed individual is a "less risky agent" (strictly speaking individual risk is associated with the law of motion of ε_i).

- **Definition 1** *Any society* **I** *can be characterized by a risk ratio schedule* $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ *with* $\lambda_i \equiv \frac{\sigma_i^2}{\sigma_{i-1}^2}$ *for* $i \in \mathbf{I}$. λ_i *refered to a "risk ratio".*

- Each individual has the same CARA utility function

$$U_i = -\frac{1}{\alpha} e^{-\alpha c_i}$$

  where c_i denotes private consumption and α a positive parameter measuring the absolute risk aversion.

- There are no financial markets. There is perfect information.

2.2 Risk-Sharing Groups.

- Individuals have an incentive to form groups in order to share risk. The motivation for individuals to form groups is that they provide insurance. More precisely, groups can be seen as mutuals as the individuals will share equally their risks. (see section Townsend in version8).

- The insurance rule we consider : agent i pays a real value of ε_i in a common fund. Because ε_i can be positive, some agents pay positive transfers; when ε_i is negative people receive transfers. (Ray, 1998, p.597)

- This insurance rule is different from credit as transfers paid at a date t do not depend on history, for instance agents who had suffered from bad shocks in the past and who received transfers do not have to reimburse them while they benefit from a good shock. (see Ray, 1998)

- This insurance rule is optimally supported when some specific weights to individual welfares are used in the social value function see APPENDIX Townsend.

- Consider a group $S \subseteq I$, formed of a finite number $n \leq N$ of agents. The expected utility of individual i in group S, $V_i(S)$, with a mutuality rule is:

$$V_i(S) = -E\left[\frac{1}{\alpha} e^{-\alpha w_i - \alpha \frac{\sum_{k \in S} \varepsilon_k}{n}}\right]$$

$$V_i(S) = -E\left[\frac{1}{\alpha} e^{-\alpha w_i - \alpha \frac{\sum_{k \in S} \varepsilon_k}{n}}\right]$$

  and, using the Arrow-Pratt formula, this can be expressed as:

$$V_i(S) = -\frac{1}{\alpha} e^{-\alpha\left[w_i - \frac{\alpha}{2n^2} \sum_{k \in S} \sigma_k^2\right]}$$

- We define the certainty-equivalent income for individual i in group S, denoted by $\omega_i\left(S\right)$, as:

$$\omega_i\left(S\right) = w_i - \frac{\alpha}{2} \sum_{k \in S} \frac{\sigma_k^2}{n^2}$$

  The risk premium for individual i in group S, denoted by $\pi_i(S)$, is equal to $\frac{\alpha}{2} \sum_{k \in S} \frac{\sigma_k^2}{n^2}$. It is immediate to remark that it is the same for every members of S. Thus, there is perfect risk-sharing between members within a group.

- The formation of a group relies on a trade-off. Accepting a new member has two effects: on the one hand, everything else equal, the size of the group is increased and the higher its size, the lower the risk premium; on the other hand, accepting an individual with a higher risk increases the risk premium for all members of the group. Therefore when assessing the net benefit of accepting a given individual, characterized by a certain variance, an insider has to weigh these two effects. But clearly, given the choice between two agents, any insider prefers the one with the lesser variance. Remark that the risk premium is a non linear function of the size n_j.

- We define the individual gain for individual i from membership to group S rather than to group S' as follows:

$$\pi_i(S') - \pi_i(S)$$

  It amounts to the reduction in the risk premium represented by being a member of S rather than a member of S'. In other words, an agent prefers joining a group (provided she is accepted in this group) in which her certainty-equivalent income is higher.

- In case of autarky ($S' = \{i\}$), this becomes:

$$\pi(\{i\}) - \pi(S).$$

Considering two individuals i and $i' > i$, $i, i' \in S$, i' benefits more than i from being in S rather than being alone:

$$\pi(\{i'\}) - \pi(S) = \frac{\alpha}{2}\left(\sigma_{i'}^2 - \sum_{k \in S} \frac{\sigma_k^2}{n^2}\right) > \pi(\{i\}) - \pi(S) = \frac{\alpha}{2}\left(\sigma_i^2 - \sum_{k \in S} \frac{\sigma_k^2}{n^2}\right)$$

Given two different risk-sharing groups, differing by their membership, and therefore, the exposure to risk of their members, the gains for joining either of them for a given agent differ. The desire of each agent is to join the group which generates for him the highest gain.

3 Risk-sharing and the segmentation of society.

The society is segmented into risk-sharing groups or coalitions voluntarily formed. Each agent is characterized by her own deterministic component and variance w_i, ε_i. This pair is perfectly known by all agents in the economy. No uncertainty on who's who.

A coalition aims at sharing risk among its members. It thus may be called a "risk-sharing coalition".

As agents have different needs for risk sharing, and expose the members of the group they join to a specific risk, the characteristics and in particular the size of a coalition matters for its members. Hence agents have different views, and maybe opposite views on possible coalitions. Here we want to endogenize the formation of risk-sharing coalitions. We do not impose either the number nor the size nor any specific conditions on coalitions. We just impose that they are voluntarily formed: no one can be imposed into a coalitions nor compelled to stay in a given coalition; to be accepted in a coalition, there must be unanimous consent by all existing members of this coalition.

We consider the following coalition-formation sequential game:

1. Agents voluntarily form risk-sharing coalitions and a partition of society is obtained.
2. The values of the idiosyncratic shocks are realized.
3. Transfers are made according to the mutuality rule in each coalition. Agents consume their after-transfer income.

Two remarks:

- Importantly, it is assumed full commitment. No agent is able to renege on her chosen coalition once the state of nature is realized. Key difference with Jenicot and Ray.

- A static game. However, remark that there is no reason to change the coalition structure between two states of nature as there is no accumulation process.
- The coalition formation process, based on voluntary moves, means that all agents belonging to a given group must unanimously agree on the inclusion of an agent willing to enter this group.

3.1 The core partition.

Given what we said on the unanimous consent to the entry of a new agent in a coalition, this formally considers to the search of a partition that belongs to the core of the coalition-formation game.

We shall need the following definitions.

Definition 2 *A non-empty subset* S_j *of* $\mathbf{I}$ *is called a group and* $\mathcal{P} = \{S_1, ..., S_j, ..., S_J\}$ *for* $j = 1, ..., J$ *is called a partition of* $\mathbf{I}$ *if: (i)* $\bigcup\limits_{j=1}^{J} S_j = \mathbf{I}$;
(ii) $S_j \bigcap S_{j'} = \emptyset$ *for* $j \neq j'$.

The size of the $j-th$ group, $S_j \subseteq \mathbf{I}$, is denoted by n_j.

Definition 3 *A partition* $\mathcal{P}^* = \left\{S_1^*, ..., S_j^*, ..., S_J^*\right\}$ *belongs to the core of the coalition-formation game if:*

$$\nexists \pounds \subseteq \mathbf{I} \text{ such that } \forall i \in \pounds,\ V_i(\pounds) > V_i(\mathcal{P}^*)$$

where $V_i(\mathcal{P}^*)$ *denotes the utility for agent* i *associated with partition* $\mathcal{P}^*$.

Then we are able to offer the following:

Proposition 1 *A core partition* $\mathcal{P}^* = \left\{S_1^*, ..., S_j^*, ..., S_J^*\right\}$ *exists and is characterized as follows:*

- *It is consecutive, that is, if* i *and* $\widetilde{i}$ *both belong to* S_j^* *then* $\forall i'$, $i > i' > \widetilde{i}$, $i' \in S_j^*$.
- *It is unique if*

$$\forall z = 2, ..., N-2, \quad \frac{\lambda_{z+1} - \lambda_z}{\lambda_{z+1} - 1} \geq -\frac{1}{z-1}. \tag{1}$$

- *For any two individuals* $i \in S_j^*$ *and* $i' \in S_{j'}^*$*, such that* $\sigma_i^2 < \sigma_{i'}^2$*, the risk premium* $\pi(S_j^*) < (=)\pi(S_{j'}^*)$ *if* $j' \neq (=)j$.

Proof. See Appendix. ■

Comments on Proposition 1:

- Existence. Agents are able by themselves to form groups so as to share risk. No institutional constraint is involved. The fact that there is no financial markets do not mean that there is no way to get insured against risk. Remark that the partition is Pareto-optimal (for this insurance rule) as we are focusing on the core.

- Consecutivity. Groups are homogeneous: they include agents who are "close" in terms of exposure to risk. As we have seen, any individual is willing to form a coalition with some one who is the less risky possible, but is not willing to form a coalition with some one who is "too" risky. Hence any one wants to join a coalition with the less risky available agent. On the other hand, if this agent has to choose between two agents, he always prefers to form a coalition with the less risky of the pair. This implies that if an agent is willing to form a coalition with some other agent, then all agents who represent intermediate risk also belong to their coalition. [1]

- Given this property, from now on, we adopt the following convention that for any ,S_j^* and $S_{j'}^*$, $j' > j$ when $\sigma_i^2 < \sigma_{i'}^2$, $\forall i \in S_j^*, \forall i' \in S_{j'}^*$.

- The condition on uniqueness depends on the rank of individuals. It is a sufficient condition to obtain uniqueness. Would it be possible to find a weaker condition? The condition may appear stringent when $\lambda_z > \lambda_{z+1}$. The expression $-\frac{1}{z+1}$ is an increasing function of z which equals $\frac{-1}{3}$ when $z = 2$ and 0 when $z = N - 1$ with N large.

Given the consecutivity property, a core partition can be characterized by a series of "pivotal agents", that is agents who are the most risky agents of the coalition they belong to.

Definition 4 *Given the coalition S_j^* in the core-partition, the pivotal agent associated with S_j^* ...*

Then we are able to offer the following:

Proposition 2 *The core partition is characterized by a series of pivotal agents, characterized by .... .*

Proof. See Appendix. ■

Let us remark that the definition of a pivotal agent is independent from her absolute exposure to risk (her own variance), nor on the degree of risk aversion. What matters in the formation of a coalition, is the relative exposure to risk, that the ratio of variances.

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[1] The consecutivity property is also obtained in Henriet and Rochet (1991) but it is with respect to probability of accident but not with respect to the variance of shocks.

3.2 Particular risk-ratio schedules.

Proposition 3 *If the risk ratio schedule* $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ *is such that:*

- $\lambda_i = \lambda, \forall i \in \mathbf{I}$, *then* $n_j^* = n, \forall j = 1, ..., J-1$*;*
- $\lambda_i \leq \lambda_{i+1}, \forall i \in \mathbf{I}$, *then* $n_j^* \geq n_{j+1}^*, \forall j = 1, ..., J-1$*;*
- $\lambda_i \geq \lambda_{i+1}$, $\forall i \in \mathbf{I}$, *then* $n_j^* \leq n_{j+1}^*, \forall j = 1, ..., J-1$*.*

Proof. See Appendix. ■

3.3 Comparing stochastic distributions and risk sharing groups.

Definition 5 *For two societies* $\mathbf{I}$ *and* $\mathbf{I}'$ *with both* N *individuals, society* $\mathbf{I}$ *is weakly less (more) segmented than a society* $\mathbf{I}'$ *if the* $j-th$ *group's size in the core partition* $\mathcal{P}^*$ *associated with* $\mathbf{I}$ *is at least (at most) equal to the* $j-th$ *group's size in the core partition* $\mathcal{P}'^*$ *associated with* $\mathbf{I}'$*, for any* $j < J$*.*

This definition also implies that the number of non-residual groups in the core partition $\mathcal{P}^*$, $J-1$, is at most (at least) equal to the number of non-residual groups in the core partition $\mathcal{P}'^*$ $J'-1$.

Proposition 4 *For two societies* $\mathbf{I}$ *and* $\mathbf{I}'$,

(i) if $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ *and* $\Lambda' = \{\lambda_2', \lambda_3', ..., \lambda_N'\}$ *are such that* $\lambda_i \geq \lambda_{i+1}, \lambda_i' \geq \lambda_{i+1}'$ *and* $\lambda_i < \lambda_i'$ *whatever* $i = 2, ..., N$*, then society* $\mathbf{I}$ *is weakly less segmented than* $\mathbf{I}'$;

(ii) if $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ *and* $\Lambda' = \{\lambda_2', \lambda_3', ..., \lambda_N'\}$ *are such that* $\lambda_i < \lambda_{i+1}, \lambda_i' < \lambda_{i+1}'$ *and* $\lambda_i < \lambda_i'$ *whatever* $i = 2, ..., N$*, then society* $\mathbf{I}'$ *may be weakly less segmented than* $\mathbf{I}$.

Proof. See Appendix. ■

- From Henriet-Rochet

  This proposition stresses the crucial role of the individual risk ratios in determining the degree of social segmentation. We thus see that it is not the individual risk, σ_i^2, per se that matters for social segmentation but rather the gap, λ_i, between individual risk ratios. For instance, let us consider two societies $\mathbf{I}$ and $\mathbf{I}'$ associated respectively with Λ and Λ' such that $\lambda_i \geq \lambda_{i+1}, \lambda_i' \geq \lambda_{i+1}'$ and $\lambda_i < \lambda_i'$ whatever $i = 2, ..., N$. This condition allows that $\sigma_i'^2 < \sigma_i^2$ whatever $i = 1, ..., N$, that is ε_i' second-order stochastically dominates ε_i. Despite the fact that society $\mathbf{I}$ is riskier than $\mathbf{I}'$, item (i) of Proposistion 4 states that it is weakly less segmented than $\mathbf{I}'$.

- Hence, Proposition 4 stresses the fact that it is impossible to determine a non ambiguous relationship between the notion of second order stochastic dominance and social segmentation.
- Attention à l'emploi du terme SSD car il est défini à parti d'une relation d'ordre. Chez nous on n'a plus équivalence entre augmentation des écarts type et SSD.
- Même remarque pour risky.

3.4 The impact of individual risk on risk sharing.

Does more individual risk lead to less aggregate risk sharing (measured through the average risk premium)?

Insight: More risk heterogeneity leads to higher individual risk premia, hence higher average risk premium. Is this reasoning true? NO!

This is true for a given group. Suppose that society is constrained to form a unique group (With our notations, I is the grand coalition). Then it is true that if for each agent, her variance increases, then the individual risk premia increase and the average risk premium increase. However this is not necessarily true when agents voluntarily form their risk-sharing coalitions. It may happen that the change in the whole core-partition leads to different risk-sharing arrangements, the outcome of which is to decrease the average risk premium.

Definition 6 *The aggregate risk premium associated with the core partition $\mathcal{P}$ is defined as:*

$$\begin{aligned}\overline{\pi}(\mathcal{P}) &= \frac{1}{N}\left(\sum_{j=1}^{J+1} n_j \pi(S_j)\right)\\ &= \frac{1}{N}\frac{\alpha}{2}\left(\sum_{j=1}^{J+1}\frac{1}{|S_j|}\sum_{k\in S_j}\sigma_k^2\right)\\ &= \frac{1}{N}\frac{\alpha}{2}\left(\sum_{j=1}^{J+1}\frac{1}{n(\lambda)}\sigma_1^2\sum_{k\in S_j}\lambda^k\right).\end{aligned}$$

Proposition 5 *For two societies $\mathbf{I}$ and $\mathbf{I}'$ with identical σ_1^2 and such that any ε_i SSDs ε_i' for $i > 1$, then society $\mathbf{I}$ may be characterized by a higher average risk premium than $\mathbf{I}'$*

$$\overline{\pi}(\mathcal{P}) > \overline{\pi}(\mathcal{P}'). \tag{2}$$

where $\mathcal{P}$ (resp. $\mathcal{P}'$) is the core partition associated with $\mathbf{I}$ ($\mathbf{I}'$).

Proof. See Appendix. ■

A proof of the impact of the distribution of idiosyncratic shocks and the impact of the (endogenous) partition of society.

1 - Example with constant lambdas. Show that if the lambda increases, it cannot lead to a decrease in the average risk-premium.

$$
\begin{aligned}
\pi(S) &= \frac{\alpha}{2}\sum_{k\in S}\frac{\sigma_k^2}{|S|^2} = \frac{\alpha}{2}\frac{\sum_{k\in S}\sigma_k^2}{\sum_{k\in I}\sigma_i^2}\frac{\sum_{k\in I}\sigma_i^2}{|S|^2} = \frac{\alpha}{2}\frac{\sum_{k\in I}\sigma_i^2}{|S|^2}\frac{\sum_{k\in S}\sigma_k^2}{\sum_{k\in I}\sigma_i^2} \\
&= \frac{\alpha}{2}\frac{\sum_{k\in I}\sigma_i^2}{|S|^2}\rho^2\left(C_j, C_I\right) = \frac{\alpha}{2}\frac{\sum_{k\in I}\sigma_i^2}{|S|^2}\frac{1-\lambda^{n(\lambda)}}{1-\lambda^N}
\end{aligned}
$$

If we consider a higher λ, keeping constant $\sum_{k\in I}\sigma_i^2$, we can prove that the average risk premium increases with λ. TURN TO NUMERICAL EXAMPLES!

A variant. To be checked.

Proposition 6 *For two societies* $\mathbf{I}$ *and* $\mathbf{I}'$ *with identical* σ_1^2 *and such that any* ε_i *SSDs* ε_i' *for* $i > 1$, *then*

- *society* $\mathbf{I}$ *is characterized by a lower average risk premium than* $\mathbf{I}'$ *if* $\lambda_i \geq \lambda_{i+1}, \lambda_i' \geq \lambda_{i+1}'$ *and* $\lambda_i < \lambda_i'$ *whatever* $i = 2, ..., N,$.
- *society* $\mathbf{I}$ *may be characterized by a higher average risk premium than* $\mathbf{I}'$ *if* $\lambda_i < \lambda_{i+1}, \lambda_i' < \lambda_{i+1}'$ *and* $\lambda_i < \lambda_i'$ *whatever* $i = 2, ..., N,$.

Proof. See Appendix. ■

4 Segmentation and empirical issues.

The aim of this section is to study empirical implications of endogenous group formation. In particular, considering the regression usually run to test for optimal risk sharing (Townsend, 1994), we want to examine the influence of group formation on the values of estimated coefficients. **FM: When preferences are CARA and aversion to risk is the same among individuals,FM** optimal risk sharing means that, within a group where agents are insured, individual consumption of any agent

i is equal to the average consumption and does not depend on the individual income of agent i. Formally, given a group S formed of individuals indexed by i, consider the regression of individual consumptions on individual incomes and average consumption within the group, expressed as follows:

$$c_{it}(S) = \alpha_i + \varphi_i \overline{c}_t + \zeta_i y_{it} + u_{it}, \tag{3}$$

where c_{it} is consumption of individual i, $\overline{c}_t$ is average consumption within the group, y_{it} is income of individual i, u_{it} is an error term.

The right boundaries of the partition of society into risk-sharing coalitions may not have been taken into account. Suppose the village is a closed society (which is more or less the case as far as insurance is concerned). It may not be the right cluster of agents with respect to risk. In other words, the grand coalition may not form in this society because it is too diverse in terms of risk. Within the village, there may be such an heterogeneity with respect to risk that agents willingly form smaller risk-sharing coalitions. Villagers may group into smaller "neighborhoods" or districts within the village, according to their particular exposure to risk.

Consider the case where society (the village) is partitioned into a core partition $\mathcal{P}^* = \left\{ S_1^*, ..., S_j^*, ..., S_J^* \right\}$. Consider an agent i belonging to S_j^*. According to (), the adequate regression to be run should be:

$$c_{it}(S_j^*) = \alpha_i + \varphi_i \overline{c}_{jt}^* + \zeta_i y_{it} + u_{it}, \tag{4}$$

where c_{it} is consumption of individual i, $\overline{c}_{jt}^*$ is average consumption within S_j^*, y_{it} is income of individual i, u_{it} is an error term.

So suppose that, in place of (), the following regression is considered:

$$c_{it}(I) = \alpha_i + \varphi_i \overline{c}_t + \zeta_i y_{it} + u_{it}, \tag{5}$$

where c_{it} is consumption of individual i, $\overline{c}_{st}$ is society average consumption per individual, y_{it} is income of individual i, u_{it} is an error term.

Using the fact **FM supprimer: that it is assumed here FM** $\overline{c}_{st} = \overline{y}_{st}$, with $\overline{y}_{st}$ the average income over society, one can rewrite the regression as follows:

$$c_{it}(S_{jt}) = \alpha_i + \beta_i(\overline{y}_{-it}) + \gamma_i y_{it} + u_{it}$$

with $\overline{y}_{-it} \equiv \frac{\sum\limits_{z \in I \setminus \{i\}} y_{zt}}{N-1}$, $\beta_i = \frac{N-1}{N} \varphi_i$ and $\gamma_i \equiv \frac{\varphi_i}{N} + \zeta_i$.

Hence, the formulas for the OLS estimators are:

$$\beta_i = \frac{cov\left(\overline{y}_{-i,t}, c_{it}\right) var\left(y_{it}\right) - cov\left(y_{it}, c_{it}\right) cov\left(\overline{y}_{-i,t}, y_{it}\right)}{var\left(\overline{y}_{-i,t}\right) var\left(y_{it}\right) - \left[cov\left(\overline{y}_{-i,t}, y_{it}\right)\right]^2} \tag{6}$$

$$\gamma_i = \frac{cov\left(y_{it}, c_{it}\right) var\left(\overline{y}_{-i,t}\right) - cov\left(\overline{y}_{-i,t}, c_{it}\right) cov\left(\overline{y}_{-i,t}, y_{it}\right)}{var\left(y_{it}\right) var\left(\overline{y}_{-i,t}\right) - \left[cov\left(\overline{y}_{-i,t}, y_{it}\right)\right]^2} \tag{7}$$

Some computations then lead us to have:

$$\beta_i = \frac{(\frac{n_j-1}{n_j})\sigma_v^2\sigma_i^2 + (\frac{\sum_{k\in S_j\setminus\{i\}}\sigma_k^2}{n_j(N-1)})\left(\sigma_v^2+\sigma_i^2\right)}{\sigma_v^2\sigma_i^2 + (\frac{\sum_{m\in I\setminus\{i\}}\sigma_m^2}{(N-1)^2})\left(\sigma_v^2+\sigma_i^2\right)}$$

and

$$\gamma_i = 1 - \left(\frac{n_j-1}{n_j}\right)\frac{\sigma_i^2(\sigma_v^2 + \frac{\sum_{m\in I\setminus\{i\}}\sigma_m^2}{(N-1)^2}) + \frac{\sigma_v^2}{(N-1)(n_j-1)}\sum_{k\in S_j\setminus\{i\}}\sigma_k^2}{\sigma_i^2(\sigma_v^2 + \frac{\sum_{m\in I\setminus\{i\}}\sigma_m^2}{(N-1)^2}) + \frac{\sigma_v^2}{(N-1)^2}\sum_{m\in I\setminus\{i\}}\sigma_m^2}.$$

Membership has thus a direct impact on values of coefficients β_i, γ_i and ζ_i. If the grand coalition forms, that is if $S_j^* = I$, we have $\beta_i = \frac{N-1}{N}$, $\varphi_i = 1$, $\gamma_i = \frac{1}{N}$ and $\zeta_i = 0$. However if this is not the case and society is partitioned into different risk sharing groups, then we have $\beta_i \neq 1$ and $\gamma_i \neq \frac{1}{N}$. Hence the fact that the right boundaries of risk-sharing coalitions are not taken into consideration leads to a dismissal of the theory of optimal risk sharing, based on the estimation of ([**?**]), even though it is in fact supported when the right groupings of individuals are taken into account and () are estimated, using for each individual the relevant independent variables corresponding to her risk-sharing coalition in the core partition.

However the regression () is not without merits, even when society is segmented into several risk-sharing coalitions. Indeed in some cases, it allows us to get some information about the core partition, that is the segmentation of society into risk-sharing groups, provided agents are left free to set their risk sharing arrangements as they wish. In the following, it is assumed that all agents share the same risk aversion, that $\sigma_v^2 = 0$ and the mutuality rule applies.

In such a case, we deduce from (eqvillage) that for agent i belonging to S_j:

$$\beta_i = \frac{N-1}{n_j}\frac{\sum_{k\in S_j\setminus\{i\}}\sigma_k^2}{\sum_{m\in I\setminus\{i\}}\sigma_m^2}$$

and

$$\gamma_i = \frac{1}{n_j}.$$

It is therefore obvious that all agents in the same club S_j^* are characterized by the same value for γ_i. Then, the larger is a club, the lower is the value of γ_i for its members. This implies that individuals in a larger club are able to get more risk sharing as their individual consumptions depend less from their individual income.

Moreover, for a given income distribution, we show that individual consumption is more dependent of individual income, the deeper is risk heterogeneity between two succeeding individuals, i.e. higher λ_i. This comes from Proposition 3: when $\lambda_i \lesseqgtr \lambda_{i+1}$ for $i = 1, ..., N-1$, then $\gamma_i \gtreqless \gamma_{i+1}$. This is due to the result that the more heterogenous individuals are with respect to idiosyncratic shocks, the less they are willing to share risk within the same coalition.

As we have

$$cov(c_{it}, \overline{y}_t) = \frac{\sum_{k \in S_j} \sigma_k^2}{n_j N},$$

the computation of $cov(c_{it}, \overline{y}_t) \times (\gamma_i)^{-1}$ leads to a measure of the risk premium and allows to deduce the index j of the club to which individual i belongs to. Then, thanks to the consecutive property, we obtain some information about the magnitude of σ_i^2.

Finally a more segmented society weakly leads to a higher γ_i and a higher $\sum\limits_{i=1}^{N} \gamma_i = J$ for all agents i living in this society, reflecting the fact that the extent of insurance is done on a lower scale.

5 Conclusion

The importance of non-financial risk sharing arrangements. Especially true in underdeveloped economies. The village economy. But also in developed economies: financial markets are not complete. How are designed the pools of risk sharing? What are their properties and consequences?

What we obtain:

1. A characterization of the optimal segmentation of society with respect to risk, depending on the differentiated idiosyncratic risks borne by individuals.
2. A possible explanation of some empirical puzzles. Some directions for empirical studies.

Extensions

1. Other insurance rules. Likely to generate more complex arrangements and no so clear-cut results.
2. Varying risk aversion parameters.
3. The distinction between economies with financial markets and without is too crude. Even in economies with financial markets, some form of coalitions exist. Informal risk sharing

arrangements, coalitions to obtain credit, venture capital. The use of coalition theory to investigate these issues.

4. The no-commitment issue.
5. Consequences on the link between risk-sharing and growth. Here no accumulation process.

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6 Appendix.

6.1 Proof of Proposition 1.

Existence. Given the value of $V_i\left(S_j\right)$, if for the two groups S_j and $S_{j'}$, we have:

$$V_i\left(S_j\right) \geq V_i\left(S_{j'}\right) \Longleftrightarrow \frac{\alpha^2}{2\left(n_j^2\right)} \sum_{k \in S_j} \sigma_k^2 \leq \frac{\alpha^2}{2\left(n_{j'}^2\right)} \sum_{k \in S_{j'}} \sigma_k^2$$

then we have:

$$V_{i'}\left(S_j\right) \geq V_{i'}\left(S_{j'}\right), \forall i' \in I.$$

This implies that the common ranking property is satisfied, that is:

$$\forall i, k \in \mathbf{I},\ V_i\left(S_j\right) \geq V_i\left(S_{j'}\right) \Leftrightarrow V_k\left(S_j\right) \geq V_k\left(S_{j'}\right).$$

According to Banerjee *et al.* (2001), the common ranking property implies that a core partition exists.

Proof of (i): Consecutivity.

By contradiction, let us consider a core-partition $\mathcal{P}^*$ characterized by some non consecutive groups, that is, there exist individual $i, \widetilde{i} \in S_j^*$ and $i' \in S_{j'}^*$ with $i < i' < \widetilde{i}$.

Suppose first that $\pi(S_j^*) \geq \pi(S_{j'}^*)$. As $i < i' < \widetilde{i} \Longleftrightarrow \sigma_i^2 < \sigma_{i'}^2 < \sigma_{\widetilde{i}}^2$, we have $\pi(S_{j'}^*) > \pi((S_{j'}^* \backslash \{i'\}) \cup \{i\})$, which leads to

$$\forall z \in (S_{j'}^* \backslash \{i'\}) \cup \{i\},\ V_z((S_{j'}^* \backslash \{i'\}) \cup \{i\}) > V_z(\mathcal{P}^*).$$

Second, assume that $\pi(S_{j'}^*) \geq \pi(S_j^*)$. We have $\pi(S_j^*) > \pi((S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\})$, which leads to

$$\forall z \in (S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\},\ V_z((S_j^* \backslash \{\widetilde{i}\}) \cup \{i'\}) > V_z(\mathcal{P}^*).$$

Hence a contradiction with the fact that $\mathcal{P}^*$ is assumed to be a core-partition.

Proof of (ii): Uniqueness.

Let us define p_j the most risky agent of the consecutive group $S_j \backslash \{p_j\}$ with size n_j satisfying the two following inequalities:

$$\sigma_{p_j}^2 \leq [2n_j + 1] \sum_{k \in S_j \backslash \{p_j\}} \frac{\sigma_k^2}{n_j^2}$$

and

$$\sigma_{p_j+1}^2 > [2n_j + 3] \sum_{k \in S_j} \frac{\sigma_k^2}{(n_j + 1)^2}.$$

Let us consider the consecutive group S_j whose lowest-individual-risk agent is i. Given the definition of the most risky agent, we can introduce the two following functions: $\Gamma(n) = \frac{n}{2n+1}$ and $\Theta(i,n) = \frac{1}{n}\frac{\sum_{k=i}^{i+n-1}\sigma_k^2}{\sigma_{i+n}^2}$ with $n = 1, ..., N-i+1$. Let us denote $n^*(i)+1$ the size of group S_j such that:

$$\Gamma(n^*(i)) \leq \Theta(i, n^*(i))$$

and

$$\Gamma(n^*(i)+1) > \Theta(i, n^*(i)+1)$$

It is easy to check that $\Gamma(n)$ is an increasing function of n and $\Gamma(1) = \frac{1}{3}$. Given $\Theta(i,1) = 1 > \Gamma(1)$, if $\Theta(i,n)$ is decreasing with respect to n whatever $i \in I$ and $n \leq N-i$, then $n^*(i)$ is unique as $\Gamma(n) \leq \Theta(i,n)$ for $n \leq n^*(i)$ and $\Gamma(n) > \Theta(i,n)$ for $n > n^*(i)$.

The function $\Theta(i, n(i))$ is decreasing if and only if:

$$\Delta\Theta(i,n) \equiv \Theta(i,n(i)+1) - \Theta(i,n(i)) = \frac{1}{n+1}\frac{\sigma_{i+n}^2 + \sum\limits_{k=i}^{i+n-1}\sigma_k^2}{\sigma_{i+n+1}^2} - \frac{1}{n}\frac{\sum\limits_{k=i}^{i+n-1}\sigma_k^2}{\sigma_{i+n}^2} < 0 \Longleftrightarrow$$

$$\psi(i,n) = n\sigma_{i+n}^2 - \left((n+1)\frac{\sigma_{i+n+1}^2}{\sigma_{i+n}^2} - n\right)\left(\sum_{k=i}^{i+n-1}\sigma_k^2\right) < 0.$$

Let us consider the function $\psi(i,n)$. It is negative for all i, $n \leq N-i$ if

$$\psi(i,1) = \sigma_{i+1}^2 - \left(2\frac{\sigma_{i+2}^2}{\sigma_{i+1}^2} - 1\right)\left(\sigma_i^2\right) \leq 0 \text{ and } \Delta\psi(i,n) \equiv \psi(i,n+1) - \psi(i,n) \leq 0.$$

Defining $\lambda_{i+1} = \frac{\sigma_{i+1}^2}{\sigma_i^2}$, the inequality $\psi(i,1) \leq 0$ is equivalent to

$$\frac{\left(\frac{\sigma_{i+1}^2 - \sigma_i^2}{\sigma_i^2}\right) - \left(\frac{\sigma_{i+2}^2 - \sigma_{i+1}^2}{\sigma_{i+1}^2}\right)}{\left(\frac{\sigma_{i+2}^2 - \sigma_{i+1}^2}{\sigma_{i+1}^2}\right)} = \frac{\lambda_{i+1} - \lambda_{i+2}}{\lambda_{i+2} - 1} \leq 1 \tag{8}$$

It is easy to check that $\forall n \geq 1$, $\Delta\psi(i,n) \leq 0$ as

$$\Delta\psi(i,n) = ((n+1)\lambda_{i+n+1} - (n+2)\lambda_{i+n+2} + 1)(\sigma_{i+n}^2 + \left(\sum_{k=i}^{i+n-1}\sigma_k^2\right)) \leq 0 \Leftrightarrow$$

$$\frac{\lambda_{i+n+1} - \lambda_{i+n+2}}{\lambda_{i+n+2} - 1}(n+1) \leq 1.$$

Defining $z \equiv i+n+1$, we can rewrite this inequality as follows:

$$\frac{\lambda_z - \lambda_{z+1}}{\lambda_{z+1} - 1}(z-1)(\frac{z-i}{z-1}) \leq 1$$

As $\frac{(z-i)}{(z-1)} = \frac{n+1}{n+i} \leq 1$, we deduce that if for all $z = 3, ..., N-1$, $\frac{\lambda_z - \lambda_{z+1}}{\lambda_{z+1}-1}(z-1) \leq 1$, then $\triangle\psi(i,n) \leq 0$.

Given equation (8), we deduce that

$$\text{If for all } z=2,...,N-1, \frac{\lambda_z-\lambda_{z+1}}{\lambda_{z+1}-1}(z-1)\leq 1 \text{ then } \triangle\psi(i,n)\leq 0 \text{ and } \psi(i,2)\leq 0.$$

Hence, when for all $z=2,...,N-1$, $\frac{\lambda_z-\lambda_{z+1}}{\lambda_{z+1}-1}(z-1)\leq 1$, we deduce that there is a unique size n_j for the club S_j.

For extension to global unicity, use Lemma 1. See below.

Proof of (iii): Risk premium ordering.

Consider the first group S_1^*. Let us define the group $\pounds_j=\left\{1,....,n_j^*\right\}$ which is consecutive, comprised of the lowest-individual-risk agents and has the same size as group S_j^*. From the definition of the core-partition, we know that, $\forall\pounds\subset I$, $\forall z\in S_1^*$ and $\pounds$, $V_z(S_1^*)\geq V_z(\pounds)$ and in particular $\forall z\in S_1^*$ and $\pounds_j$, $\forall j=2,...,J$, $V_z(S_1^*)>V_z(\pounds_j)$ which means that $\forall\pounds_j$, $\pi(S_1^*)<\pi(\pounds_j)$. Moreover, given the consecutivity property, it is easy to show that $\pi(\pounds_j)<\pi(S_j^*)$, $\forall j>1$. Hence, $\pi(S_1^*)<\pi(S_j^*)$. Considering the population $I\backslash(S_1^*\cup S_2^*\cup...\cup S_j^*)$, the same argument can be applied for S_{j+1}^* leading to the result $\pi(S_1^*)<\pi(S_2^*)<\pi(S_3^*)<...<\pi(S_j^*)<...<\pi(S_{J-1}^*)$.

This completes the proof.

6.2 Proof of Proposition 3.

Let us first denote $S^c(i)$ any consecutive group whose less risky individual is i. We will denote by $\widehat{n}(i)$ the size of $S^c(i)$ such that $\widehat{n}(i)=\arg\max V_i(S^c(i))$ in the subset $\mathbf{I}\backslash\{1,2,...,i-1\}$, for a risk ratio schedule Λ. Hence, $\widehat{n}(i)$ satisfies inequalities characterizing a pivotal agent:

$$\Gamma(\widehat{n}(i)-1)\leq\Theta(i,\widehat{n}(i)-1) \tag{9}$$

and

$$\Gamma(\widehat{n}(i))>\Theta(i,\widehat{n}(i)) \tag{10}$$

From Proof of Proposition 1, we know that $\Gamma(n)$ is an increasing function of n and, under some condition, $\Theta(i,n)$ decreases with respect to n. We can rewrite $\Theta(i,n)$ as follows: $\frac{1}{n}\frac{\sum\limits_{k=i}^{i+n-1}\sigma_k^2}{\sigma_{i+n}^2}$

$$\Theta(i,n)=\frac{1}{n}\sum_{v=i}^{i+n-1}\prod_{z=v+1}^{i-1+n}\frac{1}{\lambda_z}$$

$\Theta(i,n)$ is a function of i such that:

(i) When $\lambda_z=\lambda$, $\forall z\in I$, then $\Theta(i,n)=\Theta(i',n)$ $\forall i,i'$.

(ii) When $\lambda_z\leq\lambda_{z+1}$, $\forall z\in I$, then $\Theta(i,n)\geq\Theta(i',n)$ for $i<i'$.

(iii) When $\lambda_z\geq\lambda_{z+1}$, $\forall z\in I$, then $\Theta(i,n)\leq\Theta(i',n)$ for $i<i'$.

Hence, items (i), (ii), (iii) and inequalities (9) and (10) lead to Proposition 3.

6.3 Proof of Proposition 4.

We will denote by $\widehat{n}(i|\,\Lambda)$ the size of $S^c(i)$ such that $\widehat{n}(i|\,\Lambda) = \arg\max V_i(S^c(i))$ in the subset $\mathbf{I}\backslash\{1,2,...,i-1\}$, for a risk ratio schedule Λ.

We first offer the following Lemma

Lemma 1 *For two societies I and I' characterized respectively by $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ and $\Lambda' = \{\lambda_2', \lambda_3', ..., \lambda_N'\}$ with $\lambda_z < \lambda_z'$ for $z = 2, ..., N$, we have $\widehat{n}(i|\,\Lambda) \geq \widehat{n}(i|\,\Lambda')$.*

Let us denote $\Theta(\overrightarrow{\lambda}_{i,n}) \equiv \Theta(i,n) = \frac{1}{n}\sum\limits_{v=i}^{i+n-1}\prod\limits_{z=v+1}^{i-1+n}\frac{1}{\lambda_z}$ with $\overrightarrow{\lambda}_{i,n} = (\lambda_{i+1}, \lambda_{i+2}, ..., \lambda_{i+n-1})$. Hence for two vectors $\overrightarrow{\lambda}_{i,n}$ and $\overrightarrow{\lambda'}_{i,n}$ where $\lambda_z' > \lambda_z$, $\forall z = i+1, ..., i+n-1$, we have $\Theta(\overrightarrow{\lambda}_{i,n}) > \Theta(\overrightarrow{\lambda'}_{i,n})$, $\forall i \in I$ and $\forall n = 1, ..., N-i+1$. Given inequalities (9) and (10) and that $\Theta(\overrightarrow{\lambda}_{i,n}) > \Theta(\overrightarrow{\lambda'}_{i,n})$, it is thus easy to deduce that the optimal size of the consecutive group begining with agent i is smaller under $\Lambda = \{\lambda_2, \lambda_3, ..., \lambda_N\}$ than under $\Lambda' = \{\lambda_2', \lambda_3', ..., \lambda_N'\}$. Hence, Lemma 1.

Assume that Λ and Λ' satisfy the following property: $\lambda_i \geq \lambda_{i+1}$, $\lambda_i' \geq \lambda_{i+1}'$ and $\lambda_i < \lambda_i'$ whatever $i = 2, ..., N$. Let us denote by $p_1(\Lambda)$, respectively $p_1(\Lambda')$, the pivotal agent of group S_1^*, respectively $S_1^{*\prime}$, in the core-partition $\mathcal{P}^*$, respectively $\mathcal{P}^{*\prime}$, associated with Λ, respectively Λ'. Given Lemma 1, we know that $p_1(\Lambda) \geq p_1(\Lambda')$. Hence, S_1^* is larger than $S_1^{*\prime}$. Let us now consider S_2^* and $S_2^{*\prime}$. Given Lemma 1, we have $\widehat{n}(p_1(\Lambda')+1|\,\Lambda) \geq \widehat{n}(p_1(\Lambda')+1|\,\Lambda')$. According to Proof of Proposition 3, we know that $\widehat{n}(i|\,\Lambda) \leq \widehat{n}(i+1|\,\Lambda)$, $\forall i \in I$ and $\widehat{n}(i|\,\Lambda') \leq \widehat{n}(i+1|\,\Lambda')$, $\forall i \in I'$. We are thus able to deduce that $\widehat{n}(p_1(\Lambda)+1|\,\Lambda) \geq \widehat{n}(p_1(\Lambda')+1|\,\Lambda) \geq \widehat{n}(p_1(\Lambda')+1|\,\Lambda')$. Hence, S_2^* is larger than $S_2^{*\prime}$. Iterating the argument until S_{J-1}^* and $S_{J-1}^{*\prime}$ leads to item (i) of Proposition 4.

When Λ and Λ' satisfy the following property: $\lambda_i < \lambda_{i+1}$, $\lambda_i' < \lambda_{i+1}'$ and $\lambda_i < \lambda_i'$ whatever $i = 2, ..., N$. From Lemma 1, we still have $p_1(\Lambda) \geq p_1(\Lambda')$ and that S_1^* is larger than $S_1^{*\prime}$. Let us now consider S_2^* and $S_2^{*\prime}$. Given Lemma 1, we still have $\widehat{n}(p_1(\Lambda')+1|\,\Lambda) \geq \widehat{n}(p_1(\Lambda')+1|\,\Lambda')$. However, from Proof of Proposition 3, we have that $\widehat{n}(i|\,\Lambda) \geq \widehat{n}(i+1|\,\Lambda)$, $\forall i \in I$ and $\widehat{n}(i|\,\Lambda') \geq \widehat{n}(i+1|\,\Lambda')$, $\forall i \in I'$. Hence, $\widehat{n}(p_1(\Lambda)+1|\,\Lambda) \leq \widehat{n}(p_1(\Lambda')+1|\,\Lambda)$ and $\widehat{n}(p_1(\Lambda')+1|\,\Lambda) \geq \widehat{n}(p_1(\Lambda')+1|\,\Lambda')$. We are thus not able to compare $\widehat{n}(p_1(\Lambda)+1|\,\Lambda)$ and $\widehat{n}(p_1(\Lambda')+1|\,\Lambda')$. This proves item (ii) of Proposition 4.

6.4 Townsend

- Townsend's program

$$\max U = \sum_{i=1}^{N}\lambda_i\left(-E_0\frac{1}{\alpha_i}\sum_{t=1}^{T}\delta^{t-1}e^{-\alpha_i c_{it}}\right) \qquad \sum_{i=1}^{N}\lambda_i = 1 \tag{11}$$

with λ_i the weight of each individual. The resource constraint :

$$\sum_{i\in I} c_{it}^I\left(\epsilon_t^I\right) = N\overline{c}_t^I\left(\epsilon_t^I\right) = NY^I\left(\epsilon_t^I\right) = \sum_{i\in I} w_{it} + N\upsilon_t + \sum_{i\in I}\varepsilon_{it} \tag{12}$$

with $\epsilon^I = (\upsilon, \varepsilon_1, \ldots, \varepsilon_j, \ldots \varepsilon_N)$. We denote by $\mu\left(\epsilon_t^I\right)$ the Lagrange multiplier associated to this constraint.

The first order conditions are :

$$\lambda_i \delta^{t-1} e^{-\alpha_i c_{it}\left(\epsilon_t^I\right)} = \lambda_k \delta^{t-1} e^{-\alpha_k c_{kt}\left(\epsilon_t^I\right)} = \mu\left(\epsilon_t^I\right) \tag{13}$$

Using the first order conditions (13) and $\sum_{i\in I} c_{it}^I\left(\epsilon_t^I\right) = N\overline{c}_t^I\left(\epsilon_t^I\right)$, we have

$$c_{it}\left(\epsilon_t^I\right) = \frac{1}{\alpha_i}\left[\ln\lambda_i - \frac{\sum_{k\in I}\frac{\ln\lambda_k}{\alpha_k}}{\sum_{k\in I}\frac{1}{\alpha_k}}\right] + \frac{\frac{1}{\alpha_i}N}{\sum_{k\in I}\frac{1}{\alpha_k}}\overline{c}_t^I\left(\epsilon_t^I\right) \tag{14}$$

If individuals have the same aversion to risk, *i.e.,* $\alpha_i = \alpha$, (14) becomes

$$c_{it}\left(\epsilon_t^I\right) = \frac{1}{\alpha}\left[\ln\lambda_i - \frac{\sum_{k\in I}\ln\lambda_k}{N}\right] + \overline{c}_t^I\left(\epsilon_t^I\right). \tag{15}$$

Consumption of individual i depends on aggregate consumption, individual weights and aversion to risk. Remark also that individual consumption does not depend on individual income but on aggregate consumption.

It is important to stress that empirical conclusions obtained by Townsend's work do not depend on the definition of a particular risk sharing rule. Depending on values of individual weights λ_i, we can have different allocation rules. For instance, if $\lambda_i = \lambda_k$, $\forall i, k \in I$, and $\alpha_i = \alpha_k$ equation (15) becomes

$$c_{it}\left(\epsilon_t^I\right) = \overline{c}_t^I\left(\epsilon_t^I\right)$$

describing an egalitarian rule.

If $\ln\lambda_i = \alpha w_i$ equation (15) becomes

$$c_{it}\left(\epsilon_t^I\right) = w_i - \frac{\sum_{k\in I} w_k}{N} + \overline{c}_t^I\left(\epsilon_t^I\right) = w_i + \upsilon_t + \frac{\sum_{i\in I}\varepsilon_{it}}{N}$$

describing what we call a mutualist rule. If the risk sharing rule does not have any fundamental influence on the empirical results of Townsend then what explains why a society does not obtain complete insurance?