The work that has been done since the early eighties on the computation of equilibria using methods related to the one introduced by Scarf (1967) can be divided into three main groups. First, work that studies the existing algorithms in more detail and tries to improve them. Secondly, the investigation of the links between methods to compute economic equilibria and adjustment dynamics in economic models. Thirdly, the development of new algorithms and the "tailoring" of existing ones to specific problems.

Ad 1. A unifying framework that captures many of the algorithms that have been proposed in the seventies and the early eighties is given by Kojima and Yamamoto (1982, 1984). They have introduced the concept of a primal-dual pair of subdivided manifolds that makes it possible to describe in a uniform way for instance the algorithm of Lemke to solve the linear complementarity problem, as well as the extension of it to solve the nonlinear complementarity problem, the algorithm of Merrill, and the algorithm of van der Laan and Talman. Their framework made it also possible to develop several new algorithms. A further improvement in the efficiency of the existing algorithms, as well as the development of new algorithms, has been made possible by introducing new simplicial subdivisions and by considering different spaces (like Cartesian products of unit simplices or polytopes) on which the algorithms are applied. For this kind of work, as well as for an overview of the literature, the reader is referred to Doup (1988). See also Dang (1991) and Dai and Yamamoto (1994). Another interesting development is the possibility to apply simplicial algorithms to integer programming problems, with possible applications to economies with indivisible commodities. Seminal work has been done by Scarf (1981a, 1981b). More recently, Dang and van Maaren (1993a, 1993b) and Yang (1994) have used tools from the theory of computation of equilibria to find an integral point in a polytope.

Ad 2. As has already been remarked in Scarf (1984), page 11, "There are alternative computational methods, noticeably those of Kellog, Li, and Yorke (1977) and Smale (1976), that avoid simplicial subdivisions completely and use instead methods of differential topology. These two apparently distinct approaches have much greater similarities than might appear on the surface." Indeed, there is a close relationship between the method of Smale (1976) and the algorithm proposed in Scarf (1967). One can view the method of Smale, when started near a corner in the price simplex, basically as Scarf's algorithm applied to a triangulation with infinitesimally small grid size, or, equivalently, as the limit case of the path generated by Scarf's algorithm when the mesh size of the triangulation converges to zero. In a similar way there is a close relationship between the process given in Kamiya (1990) and Merrill's algorithm, and the process of van der Laan and Talman (1987) and their algorithm. Moreover, one can view the processes that avoid simplicial subdivisions as economic price adjustment processes, the interpretation given in Smale (1976), van der Laan and Talman (1987), and Kamiya (1990). Not only the simplicial algorithms have stong convergence properties (they converge under the conditions of Kakutani's fixed point theorem, or related conditions in terms of excess demand correspondences), but also the processes that are based on them. Since the processes can be interpreted as price adjustment processes, we therefore have the possibility to circumvent the negative results of Saari and Simon (1978) and Saari (1985) about the existence of always converging price adjustment processes. So, Smale (1976) gives a price adjustment process that converges generically when the initial price system is close to the boundary of the price simplex, Kamiya (1990) introduces a process that converges for a generic initial price system if the total excess demand function satisfies a weak boundary property, and Herings (1997) has shown that the process of van der Laan and Talman (1987) converges generically under the standard conditions on the total excess demand function. Moreover, Herings (1997) shows that the van der Laan and Talman process resembles the Walrasian tatonnement process in the Gross Substitution Case. Moreover, it is possible to generalise the above mentioned adjustment processes to cases with linear production, see van den Elzen, van der Laan, and Talman (1994), where not only prices are adjusted, but also the intensities with which certain activities are used. Furthermore, the path-following methodology can be used to study non-tatonnement processes, where along the adjustment to a competitive equilibrium, trade is possible according to a short-run equilibrium, see Herings, van der Laan, Venniker, and Talman (1997).

Ad 3. Scarf and Shoven (1984) mention two major drawbacks of the general equilibrium model as formulated by Arrow and Debreu, "The model is inadequate in its treatment of money and financial institutions, it has great difficulty in allowing for unemployed resources, ..." Therefore, it is not surprising that economists have devoted a lot of attention to both the incorporation of unemployment (models with price rigidities) and to financial markets in general equilibrium theory (models with incomplete markets). Often the computation of equilibria in these models can be made more efficient by "tailoring" the algorithm to the specific problem. This means that one adjusts the space on which the algorithm operates, exploits the mathematical properties that a certain economic model entails, and uses the economic properties of the model to develop efficient algorithms. Such computational work in models with unemployment has been done by Cornielje and van der Laan (1986) and Nguyen and Whalley (1990). Examples in models with international trade are Mansur and Whalley (1982) and van der Laan (1985), and in models with indivisible commodities Kaneko and Yamamoto (1986). For more applications, we refer to the book of Shoven and Whalley (1992).
Sometimes one has to go beyond the traditional theory on computation. For instance, in the standard model of economies with price rigidities there exists a continuum of equilibria and there are severe degeneracy problems. A path-following algorithm that indeed computes an approximation to a continuum of equilibria has been given in Herings, Talman, and Yang (1996). More clear is even the case of models with incomplete markets. There the total excess demand function does not satisfy the continuity assumptions that are usually needed when computing equilibria. This problem can be avoided by formulating the total excess demand function function on a different space, the Grassmannian manifold, see DeMarzo and Eaves (1994). Another way to avoid the problem is by not changing the space, but by using many homotopies instead of one.
This collection of homotopies has the property that if a certain homotopy in the collection is used and it reaches a point of discontinuity, then it is always possible to switch to another homotopy in the collection that is continuous at that point (although it may be discontinuous at other points), see Brown, DeMarzo, and Eaves (1996b). Related is also the work of Brown, DeMarzo, and Eaves (1996a). Another interesting example is in the field of game theory. To compute a Nash-equilibrium in an N-person non-cooperative game, Van der Laan and Talman (1982) developed an algorithm on a product space of simplices, since that space is very suitable for computations in games. Yamamoto (1993) proposed a new procedure on the same space that does not

only compute a Nash-equilibrium, but the specifics of the procedure guarantee that eventually a proper Nash-equilibrium will be found.

Brown, D.J., P.M. DeMarzo, and B.C. Eaves (1996a), "Computing Zeroes of Sections of Vector Bundles Using Homotopies and Relocalization," Mathematics of Operations Research, 21, 26-43.

Brown, D.J., P.M. DeMarzo, and B.C. Eaves (1996b), "Computing Equilibria when Asset Markets are Incomplete," Econometrica, 64, 1-27.

Cornielje, O.J.C., and G. van der Laan (1986), "The Computation of Quantity-Constrained Equilibria by Virtual Taxes," Economics Letters, 22, 1-6.

Dai, Y., and Y. Yamamoto (1994), "A Continuous Deformation Algorithm for Variational Inequality Problems on Polytopes," Mathematical Programming, 64, 103-122.

Dang, C. (1991), "The D1-triangulation of Rn for simplicial algorithms for computing solutions of nonlinear equations", Mathematics of Operations Research, 16, 148-161.

Dang, C., and H. van Maaren (1993a), "A Simplicial Approach to Integer Programming-Part I," Working Paper, Department of Mathematics, Delft University of Technology, Delft.

Dang, C., and H. van Maaren (1993b), "A Simplicial Approach to Integer Programming-Part II," Working Paper, Department of Mathematics, Delft University of Technology, Delft.

DeMarzo, P.M., and B.C. Eaves (1994), "Computing Equilibria of GEI Using the Grassmann Manifold and Relocalization," Working Paper, Stanford University. To appear in Journal of Mathematical Economics.

Doup, T.M. (1988), Simplicial Algorithms on the Simplotope, Lecture Notes in Economics and Mathematical Systems, 318, Springer-Verlag, Berlin.

Elzen, A.H. van den, G. van der Laan, and A.J.J. Talman (1994), "An Adjustment Process for an Economy with Linear Production Technologies," Mathematics of Operations Research, 19, 341-351.

Herings, P.J.J. (1997), "A Globally and Universally Stable Price Adjustment Process," Journal of Mathematical Economics, 27, 163-193.

Herings, P.J.J., G. van der Laan, A.J.J. Talman, and R. Venniker (1997), "Equilibrium Adjustment of Disequilibrium Prices," Journal of Mathematical Economics, 27, 53-77.

Herings, P.J.J., A.J.J. Talman, and Z. Yang (1996), "The Computation of a Continuum of Constrained Equilibria," Mathematics of Operations Research, 21, 675-696.

Kamiya, K. (1990), "A Globally Stable Price Adjustment Process," Econometrica, 58, 1481-1485.

Kaneko, M., and Y. Yamamoto (1986), "The Existence and Computation of Competitive Equilibria in Markets with an Indivisible Commodity", Journal of Economic Theory, 38, 118-136.

Kellog, R.B., T.Y. Li, and J. Yorke (1977), "A Method of Continuation for Calculating a Brouwer Fixed Point," in S. Karamardian (ed.), Computing Fixed Points with Applications, Academic Press, New York, 133-147.

Kojima, M., and Y. Yamamoto (1982), "Variable Dimension Algorithms: Basic Theory, Interpretations and Extensions of Some Existing Methods," Mathematical Programming, 24, 177-215.

Kojima, M., and Y. Yamamoto (1984), "A Unified Approach to the Implementation of Several Restart Fixed Point Algorithms and a New Variable Dimension Algorithm," Mathematical Programming, 28, 288-328.

Laan, G. van der (1985), "The Computation of General Equilibrium in Economies with a Block Diagonal Pattern," Econometrica, 53, 659-665.

Laan, G. van der, and A.J.J. Talman (1982), "On the Computation of Fixed Points in the Product Space of Unit Simplices and an Application to Noncooperative N Person Games," Mathematics of Operations Research, 7, 1-13.

Laan, G. van der, and A.J.J. Talman (1987), "A Convergent Price Adjustment Process," Economics Letters, 23, 119-123.

Mansur, A., and J. Whalley (1982), "A Decomposition Algorithm for General Equilibrium Computation with Application to International Trade Models," Econometrica, 50, 1547-1557.

Nguyen, T.T., and J. Whalley (1990), "General Equilibrium Analysis of Price Controls: A Computational Approach", International Economic Review, 31, 667-684.

Saari, D.G. (1985), "Iterative Price Mechanisms," Econometrica, 53, 1117-1131.

Saari, D.G., and C.P. Simon (1978), "Effective Price Mechanisms," Econometrica, 46, 1097-1125.

Scarf, H. (1967), "The Approximation of Fixed Points of a Continuous Mapping," SIAM Journal on Applied Mathematics, 15, 1328-1343.

Scarf, H. (1981a), "Production Sets with Indivisibilities-part I: Generalities," Econometrica, 49, 1-32.

Scarf, H. (1981b), "Production Sets with Indivisibilities-part II: the Case of Two Activities," Econometrica, 49, 395-423.

Scarf, H. (1984), "The Computation of Equilibrium Prices," in H.E. Scarf and J.B. Shoven (eds.), Applied General Equilibrium Analysis, Cambridge University Press, Cambridge, pp. 1-49.

Scarf, H.E., and J.B. Shoven (1984), "Preface," in H.E. Scarf and J.B. Shoven (eds.) Applied General Equilibrium Analysis, Cambridge University Press, Cambridge, pp. ix-xiii.

Shoven, J.B., and J. Whalley (1992), Applying General Equilibrium, Cambridge University Press, Cambridge.

Smale, S. (1976), "A Convergent Process of Price Adjustment and Global Newton Methods," Journal of Mathematical Economics, 3, 107-120.

Yamamoto, Y. (1993), "A Path-Following Procedure to Find a Proper Equilibrium of Finite Games", International Journal of Game Theory, 22, 249-259.

Yang, Z. (1994), "A Simplicial Algorithm for Testing the Integral Property of Polytopes," CentER Discussion Paper No. 9489, Tilburg University, Tilburg.