

A holding problem can be described by means of an  $(n + 2)$ -tuple  $\mathcal{H} = \langle N, h, B_1, \dots, B_n \rangle$ , where  $N = \{0, 1, \dots, n\}$  is the set of involved players,  $h \in \mathfrak{R}_+$  is the holding capacity of player 0, and for  $i \in \{1, \dots, n\}$  the function  $B_i : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$  is the benefit function of player  $i$ . For  $x \in \mathfrak{R}_+$ ,  $B_i(x)$  is the benefit that agent  $i$  gets if this player is allowed to store an amount  $x$ . Benefit functions are assumed to be concave, to map 0 to 0, to be constant above a certain threshold value, and to have a finite right-derivative at 0.

The aggregate benefit function of a group  $S$  is the function  $B_S : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ , defined by  $B_S(a) = \max\{\sum_{i \in S} B_i(x_i) \mid (x_i)_{i \in S} \in \mathfrak{R}_+^S \text{ and } \sum_{i \in S} x_i = a\}$ . The benefit function can be used to model the holding problem as a cooperative transferable utility game  $(N, v_{\mathcal{H}})$ , with characteristic function defined by  $v_{\mathcal{H}}(S \cup \{0\}) = B_S(h)$  and  $v_{\mathcal{H}}(S) = 0$ , for non-empty  $S \subset \{1, \dots, n\}$ . Moreover,  $v_{\mathcal{H}}(\{0\}) = 0$ .

Section 2 derives a necessary and sufficient condition for a feasible storage plan  $(z_i)_{i \in S} \in \mathfrak{R}_+^S$  to be optimal for  $S \cup \{0\}$ . A characterization of the generalized derivative of  $B_S$  is given.

Section 3 shows that  $(N, v_{\mathcal{H}})$  is a strong big boss game, with 0 as big boss, i.e.  $v_{\mathcal{H}}(S) \subset v_{\mathcal{H}}(T)$  when  $S \subset T$ ,  $v_{\mathcal{H}}(S) = 0$  if  $0 \notin S$ , and the union property that  $v_{\mathcal{H}}(N) - v_{\mathcal{H}}(S) \geq \sum_{i \in N \setminus S} M_i(v_{\mathcal{H}})$  for all  $S \subset N$  such that  $0 \in S$ , where  $M_i(v_{\mathcal{H}}) = v_{\mathcal{H}}(N) - v_{\mathcal{H}}(N \setminus \{i\})$ , the marginal contribution of player  $i$  to the grand coalition  $N$ .

From the properties of big boss games in general, a number of further properties are deduced in Section 4. The core and the bargaining set coincide, both being a parallelotope, which can be easily described. The core is a stable set if and only if the game is convex. Moreover, the  $\tau$ -value and the nucleolus coincide and they are located in the center of the core. The kernel consists of one point, which coincides with the nucleolus. The Shapley value, the nucleolus and the  $\tau$ -value coincide for convex games. It is shown that the holding problem can also be studied as an economic equilibrium problem. The set of equilibria is analyzed and is shown to be a refinement of the core. The paper concludes with a survey of numerical methods to compute an optimal storage plan.