

A finite number of players, $1, 2, \dots, n$, negotiates how to split a pie of size 1 via $(n - 1)$ bilateral bargaining sessions. In each bilateral bargaining session, two players negotiate a partial agreement via the alternating-proposal framework of Rubinstein (1982). An outcome of the bargaining problem consists of $(n - 1)$ partial agreements that specify player i 's share of the pie, denoted as $x_i \in [0, 1]$, for $i = 1, 2, \dots, n$ such that $\sum_{i=1}^n x_i = 1$, and the date $T \geq 1$ when such a full agreement is reached. From such an outcome, player i 's payoff is given by $\delta_i^{T-1} u_i(x_i)$, where $\delta_i \in (0, 1)$ is player i 's discount factor per bargaining period and $u_i(\cdot)$ is player i 's utility function which is assumed to be concave and continuously differentiable. If players disagree forever, i.e. $T = \infty$, every player receives a payoff of 0. The paper considers two bargaining procedures, an offer procedure and a demand procedure.

In the offer procedure, the proposer offers a share to the responder. Should the offer be accepted, the responder will exit the bargaining, and the proposer will meet a new responder in the next session. Should the offer be rejected, then the roles of proposer and responder are interchanged, and so on, and so forth. The offer procedure continues in this fashion for all remaining sessions. It can be shown that there is a unique subgame perfect equilibrium that is efficient. The equilibrium outcome is determined by the following two recursive relations:

$$\begin{aligned} u_j(X_i) &= \delta_j u_j(s_j^j(Y + X_j)), \\ u_i(X_j) &= \delta_i u_i(s_i^j(Y + X_i)), \end{aligned}$$

where Y denotes the total share that is offered to the players that have already agreed to exit, player i offers X_i and accepts X_j , and player j offers X_j and accepts X_i . For player $i' = i$ or j , the share of player i' after his offer $X_{i'}$ is accepted is denoted $s_{i'}^j(Y + X_{i'})$. When all players have the same discount factor δ and linear utility function, players' payoffs in the unique subgame perfect equilibrium under the offer procedure are

$$\left(\frac{1}{1 + (n - 1)\delta}, \frac{\delta}{1 + (n - 1)\delta}, \dots, \frac{\delta}{1 + (n - 1)\delta} \right).$$

In the demand procedure, the proposer demands a share from the responder. Should the demand be accepted, the proposer will exit the bargaining, and the responder becomes the proposer in the next session. Should the demand be rejected, then the roles of proposer and responder are interchanged, and so on, and so forth. The demand procedure continues in this fashion for all remaining sessions. It can be shown that there is a unique subgame perfect equilibrium that is efficient. The equilibrium outcome is determined by the following two recursive relations:

$$\begin{aligned} u_j(s_j^j(Y + X_i)) &= \delta_j u_j(X_j), \\ u_i(s_i^j(Y + X_j)) &= \delta_i u_i(X_i). \end{aligned}$$

When all players have the same discount factor δ and linear utility function, players' payoffs in the unique subgame perfect equilibrium under the offer procedure are

$$\left(\frac{1}{1 + \delta + \delta^2 + \dots + \delta^{n-1}}, \frac{\delta}{1 + \delta + \delta^2 + \dots + \delta^{n-1}}, \dots, \frac{\delta^{n-1}}{1 + \delta + \delta^2 + \dots + \delta^{n-1}} \right).$$

Moreover, it can be shown for both procedures that, if all players have the same discount factor δ , then the unique subgame perfect equilibrium outcome converges to the symmetric Nash bargaining solution of the corresponding bargaining problem as δ goes to one. This result does not require utility functions to be linear.

Rubinstein, A., (1982). Perfect equilibrium in a bargaining model. *Econometrica* 50, 97–109.