

Let $\Omega = \{\omega^i\}_{i=1}^N$ be a finite state space, and assume that the decision maker chooses among acts with state contingent payoffs $z : \Omega \rightarrow \mathbb{R}$. In the Choquet expected utility (CEU) model, an ambiguity averse decision maker's subjective belief is represented by a convex non-additive probability function ν such that $\nu(\emptyset) = 0$, $\nu(\Omega) = 1$, and $\nu(X \cup Y) \geq \nu(X) + \nu(Y) - \nu(X \cap Y)$, for all $X, Y \subseteq \Omega$. The core of ν is defined as

$$C(\nu) = \{\pi \in \Delta(\Omega) \mid \pi(X) \geq \nu(X), \text{ for all } X \subseteq \Omega\},$$

so $\nu(X) = \min_{\pi \in C(\nu)} \pi(X)$. The ambiguity of the belief about an event X is measured by the expression $\mathcal{A}(X, \nu) = 1 - \nu(X) - \nu(X^c) = \max_{\pi \in C(\nu)} \pi(X) - \min_{\pi \in C(\nu)} \pi(X)$.

Given a convex non-additive probability ν and a utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$, the Choquet expected utility of an act is

$$\mathbb{C}\mathbb{E}_\nu u(z) = \min_{\pi \in C(\nu)} \left\{ \sum_{\omega \in \Omega} u(z(\omega)) \pi(\omega) \right\} = \int_{\Omega} u(z(\omega)) d\nu.$$

The authors study a two-period monetary general equilibrium model. There are two groups of agents in the model. The first group, with agents indexed by $h = 1, \dots, H$, are those who trade on financial markets, while the second group, with agents indexed by $k = 1, \dots, K$, has no access to any financial markets. There are three goods, x , y , and z . The endowment of h -type agents is uncontingent, given by $((\bar{x}_h^0, \bar{z}_h^0), (\bar{x}_h, \bar{z}_h, \bar{m}_h))$, where $(\bar{x}_h^0, \bar{z}_h^0)$ is the endowment in the initial period, and $(\bar{x}_h, \bar{z}_h, \bar{m}_h)$ is the endowment in the final period. Since transfers should balance across households, we have

$$\sum_{h=1}^H \bar{m}_h = 0.$$

The endowment of k -type agents are given by $(\bar{y}_k^0, \bar{z}_k^0)$ in the initial period. Their final period endowment in good z is uncontingent and equal to \bar{z}_k . Their endowments in good y is contingent: in state t , \bar{y}_k^t , are such that $\sum_{k=1}^K \bar{y}_k^t = y^L$ for, say, $t = 1, \dots, \tau$ and $\sum_{k=1}^K \bar{y}_k^t = y^H$ for $t = \tau + 1, \dots, T$. There is money in the model, whose supply in Period 0 is fixed at M^0 but may take on two values in Period 1, m or M , where $M = \lambda m$, $\lambda > 1$. Agents sell to a central authority all their endowments against currency issued by the central authority and then buy back from that authority the goods they want to consume. The money obtained from the central authority by agent h (respectively, k) from the sale of endowments in state s is denoted m_h^s (respectively, m_k^s .)

Prices of goods x , y , and z in state s are denoted as p_x^s , p_y^s , and p_z^s , respectively. In Period 0 two financial assets are traded. The first is a nominal bond b^n that pays off one unit of money in all states and with price denoted q^n . The second is an indexed bond b^i that pays off one unit of each good at each state in Period 1. Its price is denoted by q^i . Agent

h 's preferences are denoted by a functional $V_h((x_h^0, z_h^0), \dots, (x_h^S, z_h^S))$. The maximization problem of household h is therefore the maximization of this functional by choosing x_h , z_h , b_h^i , and b_h^n subject to

$$\begin{aligned} p_x^0 \bar{x}_h^0 + p_z^0 \bar{z}_h^0 &= m_h^0, \\ p_x^0 x_h^0 + p_z^0 z_h^0 &= m_h^0 - q^i b_h^i - q^n b_h^n, \\ p_x^s \bar{x}_h^s + p_z^s \bar{z}_h^s + \bar{m}_h &= m_h^s \\ p_x^s x_h^s + p_z^s z_h^s &= m_h^s + b_h^n + (p_x^s + p_y^s + p_z^s) b_h^i, \quad s = 1, \dots, S. \end{aligned}$$

Agents k are assumed to solve $S + 1$ separate maximization problems, for $s \in \mathcal{S}$ given by

$$\begin{aligned} &\max_{y_k^s, z_k^s} (y_k^s)^\alpha (z_k^s)^{1-\alpha} \\ \text{s.t. } &p_y^s \bar{y}_k^s + p_z^s \bar{z}_k^s = m_k^s \\ &p_y^s y_k^s + p_z^s z_k^s = m_k^s. \end{aligned}$$

An equilibrium of the model is an allocation (x, y, z, m, b^i, b^n) and prices $(p_x, p_y, p_z, q^i, q^n)$ such that, given these prices agents solve their maximization problems and markets clear.

To facilitate tractability, further assumptions on agent's h preferences are made. At state 0, h -type agents' utility function is written as $u(x_h^0, z_h^0)$, where $u : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$ is strictly increasing, concave, differentiable and homogeneous of degree 1. In state s the spot utility function of agent h is given by $f_h(u(x_h^s, z_h^s))$, where $f_h : \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing, strictly concave and differentiable, where u is as defined above. Type- h agents are endowed with common beliefs μ about the money supply and ν on the process generating the y -endowments.

A first main result is that for generic endowments, if beliefs are not ambiguous, there will always be trade in indexed bonds.

Theorem 1. *Suppose $\mu^m + \mu^M = 1$ and $\nu^L + \nu^H = 1$. Then, for generic first period aggregate endowments, there is trade in the indexed bond whenever $\lambda \neq 1$.*

The second main result shows that, at equilibrium, if ambiguity of beliefs about the y -prices is large enough and inflation risk (λ) is not too high, the indexed bond is not traded and only the nominal bond is traded.

Theorem 2. *Suppose, $\mu^m + \mu^M \leq 1$ and $\nu^L + \nu^H < 1$. Then, there exists a bound δ , $\delta < 1$, such that, if $\lambda < \delta$, there exists γ , $0 < \gamma < 1$, such that if $\mathcal{A}(\nu) > \gamma$, then at equilibrium,*

1. *the indexed bond is not traded, i.e., $b_h^i = 0$ for all h ,*

2. *for generic μ -beliefs, there is trade in the nominal bond as long as there exists an h such that $\bar{m}^h > 0$.*

This result holds irrespective of the degree of ambiguity about the money supply.