A continuum cost allocation problem (CAP) is a pair $(z; F)$. Here $z \in \mathbb{R}^n_+ \setminus \{0\}$ is a vector of demands, and $F : [0, z] \to \mathbb{R}$ with $F(0) = 0$ is a cost function. A solution is a function that selects a vector in $\mathbb{R}^n$ for each continuum CAP.

A discrete CAP is a pair $(q; C)$. Here $q \in \mathbb{Z}^n_+ \setminus \{0\}$ is a vector of integer demands, and $C : [0, q] \cap \mathbb{Z}^n \to \mathbb{R}$ with $C(0) = 0$ is a cost function. A solution is a function that selects a vector in $\mathbb{R}^n$ for each discrete CAP.

The aim of the paper is to establish the relation between the limit of linear solutions of a discrete CAP and the solution of a continuum CAP. A continuum CAP $(z; F)$ generates a discrete CAP as follows. Let $k \in \mathbb{N}^n_+$ be given. The discrete CAP $(k; F^{z,k})$ is defined by

$$F^{z,k}(s) = F\left(\frac{s_1}{k_1}z_1, \ldots, \frac{s_n}{k_n}z_n\right), \quad \text{for all } 0 \leq s \leq k,\ s \in \mathbb{Z}^n_+.$$

A sequence $\{k^m\}_{m\in\mathbb{N}}$ is called admissible if, for all $i \in N$, $\lim_{m\to\infty} k^m_i = \infty$. The authors prove the following limit result as Lemma 1.

Let ζ be a linear solution on discrete CAP, $(z; F)$ be a continuum CAP in Ω and $i \in N$. Then

$$\lim_{m\to\infty} \zeta_i(k^m; F^{z,k^m})$$
$$= \lim_{m\to\infty} \sum_{T\subset N,\ T\ni i} \sum_{s\in B^{k^m}(T)} \frac{\partial^{|T|}F}{\partial x_T}\left((\tfrac{s_j}{k^m_j}z_j)_{j\in N}\right) \cdot \prod_{j\in T}\left(\tfrac{z_j}{k^m_j}\right) \cdot \zeta_i(k^m; C^{k^m}_s),$$

for all admissible sequences $\{k^m\}_{m\in\mathbb{N}}$. In particular,

$$\lim_{m\to\infty} \sum_{0\leq s\leq k^m} \Delta^{z,k^m}_F(s) = \sum_{T\subset N} \int_{[0,z_T]} \frac{\partial^{|T|}F}{\partial x_T}(x_T)dx_T,$$

where $\Delta^{z,k^m}_F(0) = 0$ and, for $s \neq 0$, $\Delta^{z,k^m}_F(s) = F(s) - \sum_{0\leq t<s} \Delta^{z,k^m}_F(t)$.

Using this result, Corollary 2 establishes the Shapley-Shubik solution for continuum CAP as a limit of the Shapley-Shubik solution for discrete CAP and Corollary 3 the serial cost sharing solution for continuum CAP as a limit of the serial cost sharing solution for discrete CAP. Corollary 4 obtains a new solution for continuum CAP, called pseudo-average cost method, as a limit of the pseudo-average cost method for discrete CAP.