

Ju considers a society consisting of $n \geq 2$ agents, $N = \{1, \dots, n\}$. There is a finite number S of uncertain states with $S \geq 2$. At each state $s \in S$, a fixed amount Ω_s of a single good, or money, is available in the society. The aggregate certainty case applies if $\Omega_1 = \dots = \Omega_S$. Otherwise, aggregate uncertainty holds. Individual endowments are given by $\omega_i \in \mathfrak{R}_+^S$. They sum up to Ω . Agents have a common prior π over the state space. Agents' preferences R_i take the expected utility form and are continuous, strictly monotonic and weakly convex. An economy is characterized by agents' preferences and individual endowments. The set of all economies satisfying the main assumptions is denoted by \mathcal{E} . The problem considered is the allocation of the aggregate endowment among agents before the realization of the state. A risk sharing rule φ associates with each economy a single feasible allocation, a list $(z_i)_{i \in N} \in \mathfrak{R}_+^{N \times S}$ of state-contingent bundles with sum equal to the aggregate endowment. The set of feasible allocations is denoted by Z .

Ju studies rules satisfying efficiency, individual rationality and strategy-proofness. A rule φ is efficient if for all $(R, \omega) \in \mathcal{E}$ there is no $z \in Z$ satisfying $z_i R_i \varphi_i(R, \omega)$ for all $i \in N$ and $z_j P_j \varphi_j(R, \omega)$ for some $j \in N$, where P_j denotes the strict preference relation derived from R_j . A rule is individual rational if all agents are at least as well off as in their initial endowments. Strategy-proofness requires that no one can ever benefit by misrepresenting his preference, independently of others' representations, for all $(R, \omega) \in \mathcal{E}$, for all $i \in N$, and all $R'_i \in \mathcal{R}$, $\varphi_i(R, \omega) R_i \varphi_i((R'_i, R_{-i}), \omega)$. A rule φ is dictatorial if for each profile of individual endowments, there exists an agent $i \in N$ who always receives the entire aggregate endowment Ω independently of preferences.

A Walrasian equilibrium allocation for (R, ω) is an allocation $z \in Z$ that has a price vector p such that for all $i \in N$ and all $z'_i \in \mathfrak{R}_+^S$ with $p \cdot z'_i \leq p \cdot \omega_i$, $z_i R_i z'_i$. The set of Walrasian allocations is denoted by $W(R, \omega)$. The set-valued function $W : \mathcal{E} \rightarrow Z$ is called the Walrasian correspondence. A rule is a fixed price selection from the Walrasian correspondence if there is a price vector p such that the rule maps each economy into a Walrasian allocation supported by p . A typical example is the case where the price vector p is taken equal to the common prior π .

For the case with aggregate certainty, i.e. aggregate income is constant across states, it is shown that all fixed price selections from the Walrasian correspondence are efficient, individually rational, and strategy-proof rules and, moreover, they are the only such rules. When aggregate certainty does not hold, it is shown that there exists no efficient, individually rational,

and strategy-proof rule. Therefore, aggregate certainty is a necessary and sufficient condition for the existence of rules satisfying efficiency, individual rationality, and strategy-proofness.

For the two agents case, a stronger impossibility result is derived: dictatorial rules are the only efficient and strategy-proof rules. Dropping the common prior assumption in the model, it is shown that in the two agents and aggregate certainty case, this assumption is necessary and sufficient for the existence of rules satisfying the three main requirements.

Finally, extending the model to allow for more than one good, it is shown that when there are two agents and at least two goods, dictatorial rules are the only efficient and strategy-proof rules, independently of the nature of aggregate risk.