

The paper considers production economies with  $L$  goods,  $I$  consumers (characterized by their consumption sets  $X_i$ , preferences,  $R_i$  and endowments  $w_i$ ), and  $J$  firms (characterized by their production sets  $\mathcal{Y}_j$ ). A full specification of all characteristics yields an economy  $e$ . The set of all economies is denoted by  $E$ .

An allocation  $z = (x, y) \in \mathbb{R}^{IL} \times \mathcal{Y}$  is a  $\theta$ -Walrasian allocation for an economy  $e$  if it is feasible and there is a price vector  $p \in \Delta^{L-1}$  such that

1.  $p \cdot x_i = \sum_{j=I+1}^N \theta_{ij} p \cdot y_j$  for all  $i = 1, \dots, I$ ;
2. for all  $i = 1, \dots, I$ ,  $(x'_i + w_i) P_i(x_i + w_i)$  implies  $p \cdot x'_i > \sum_{j=I+1}^N \theta_{ij} p \cdot y_j$ ; and
3.  $p \cdot y_j \geq p \cdot y'_j$  for all  $y'_j \in \mathcal{Y}_j$  and  $j = I + 1, \dots, N$ .

The set  $W_\theta(e)$  is the set of all Walrasian allocations, and the set  $\mathcal{W}_\theta(e)$  denotes all Walrasian price-allocation pairs  $(p, z)$ .

The Walrasian process  $\langle M_c, \mu_c, h_c \rangle$  is defined as follows. Define the message space  $M_c = \Delta^{L-1} \times Z$ , where  $Z$  is the set of balanced allocations, i.e. those allocations where total supply equals total demand. Define the correspondence  $\mu_c : E \rightarrow M_c$  by

$$\mu_c(e) = \cap_{i=1}^N \mu_{ci}(e_i),$$

where  $\mu_{ci} : E \rightarrow M_c$  is defined as follows

1. For  $i = 1, \dots, I$ ,  $\mu_{ci}(e_i) = \{(p, x, y) : p \in \Delta^{L-1}, x_i \in D_i(p, \theta, p \cdot y_{I+1}, \dots, p \cdot y_N, e_i) \text{ and } \sum_{i=1}^I x_i = \sum_{j=I+1}^N y_j\}$ .
2. For  $i = I + 1, \dots, N$ ,  $\mu_{ci}(e_i) = \{(p, x, y) : p \in \Delta^{L-1}, y_i \in S_i(p, e_i) \text{ and } \sum_{i=1}^I x_i = \sum_{j=I+1}^N y_j\}$ .

Then,  $\mu_c(e) = \mathcal{W}_\theta(e)$  for all  $e \in E$ .

The Walrasian outcome function  $h_c : M_c \rightarrow Z$  is defined by

$$h_c(p, x, y) = (x, y),$$

which is an element in  $W_\theta(e)$ .

In general, a mechanism is a triple  $\langle M, \mu, h \rangle$ , where  $M$  is the message space,  $\mu : E \rightarrow M$  is the equilibrium message correspondence, and  $h : M \rightarrow Z$  is the outcome function that assigns to every equilibrium message

$m \in \mu(e)$  the corresponding trade  $z \in Z$ . The allocation mechanism is said to be informationally decentralized on  $E$  if there exist individual message correspondences  $\mu_i : E_i \rightarrow M$  such that  $\mu(e) = \cap_{i=1}^N \mu_i(e_i)$ , where  $E = \prod_{i=1}^N E_i$  and  $E_i$  is the set of characteristics of individual  $i$ . Let  $\mathcal{P}(e)$  be a subset of the Pareto efficient allocations for  $e \in E$ . An allocation mechanism  $\langle M, \mu, h \rangle$  is non-wasteful on  $E$  with respect to  $\mathcal{P}$  if for all  $e \in E$ ,  $\mu(e) \neq \emptyset$  and  $h(m) \in \mathcal{P}(e)$  for all  $m \in \mu(e)$ . An allocation mechanism  $\langle M, \mu, h \rangle$  is locally threaded at  $e \in E$  if it has locally a continuous, single-valued selection at  $e$ .

An informationally decentralized non-wasteful mechanism is informationally efficient if the size of its message space  $M$  is the smallest one among all other informationally decentralized non-wasteful mechanisms.

The first result of the paper states the following. Suppose that  $\langle M, \mu, h \rangle$  is an allocation mechanism on any class of production economies  $E$  that includes  $E^{cq}$ , economies with Cobb-Douglas preferences and quadratic production possibility sets, such that

1. it is informationally decentralized;
2. it is non-wasteful with respect to  $\mathcal{P}$ ;
3.  $M$  is a Hausdorff topological space;
4.  $\mu$  is locally threaded at some point  $e \in \bar{E}^{cq}$ .

Then, the size of the message space  $M$  is at least as large as  $\mathbb{R}^{(L-1)I+LJ}$ , that is,  $M \geq_F M_c =_F \mathbb{R}^{(L-1)I+LJ}$ , where  $\geq_F$  denotes the Fréchet ordering.

The second result establishes the informational efficiency of the competitive mechanism within the class of smooth resource allocation mechanisms which are informationally decentralized and non-wasteful over the class of Walrasian production economies  $E^c$ . More precisely, the Walrasian allocation mechanism  $\langle M_c, \mu_c, h_c \rangle$  is informationally efficient among all allocation mechanisms  $\langle M, \mu, h \rangle$  defined on  $E^c$  that

1. it is informationally decentralized;
2. it is non-wasteful with respect to the Pareto efficient allocations;
3.  $M$  is a Hausdorff topological space;
4.  $\mu$  is locally threaded at some point  $e \in \bar{E}^{cq}$ .

That is,  $M_c =_F \mathbb{R}^{(L-1)I+LJ} \leq_F M$ .

The third result proves that the competitive mechanism is the unique informational efficient one. Suppose that  $\langle M, \mu, h \rangle$  is an allocation mechanism on the class of production economies  $E^{cq}$  such that

1. it is informationally decentralized;
2. it is non-wasteful with respect to  $\mathcal{P}$ ;
3. it is individually rational with respect to the fixed share guarantee structure  $\gamma_i(e; \theta)$ ;
4.  $M$  is a  $(L - 1)I + LJ$  dimensional manifold;
5.  $\mu$  is a continuous function on  $E^{cq}$ .

Then, there is a homeomorphism  $\phi$  on  $\mu(E^{cq})$  to  $M_c$  such that

1.  $\mu_c = \phi \cdot \mu$ ;
2.  $h_c \cdot \phi = h$ .

It should be noticed that agents follows the rules of the mechanism without regard to self-interest. When the incentive aspect of the Walrasian mechanism is also taken into account, Reichelstein and Reiter (1988) have shown that a Nash implementation typically increases the size of the message space of the mechanism.

Reichelstein, S., and S. Reiter (1988), "Game Forms with Minimal Strategy Spaces," *Econometrica*, 49, 661–692.