The authors consider an oligopoly with n firms selling homogeneous goods at every time $t \in [0, \infty)$. The market demand function at time t equals $p(t) = A - q_i(t) - Q_{-i}(t)$, where $Q_{-i}(t) = \sum_{j \neq i} q_j(t)$ is the output supplied by all firms other than i. Firms can produce at time t against constant marginal costs $c_i(t)$. Marginal costs evolve over time according to the equation

$$\dot{c}_i(t) = c_i(t)[-k_i(t) - \beta K_{-i}(t) + \delta],$$

where $k_i(t)$ is the R & D effort exerted by firm i at time t and $K_{-i}(t) = \sum_{j \neq i} k_j(t)$ is the aggregate $R\&D$ effort. The parameter $\beta \in [0, 1]$ reflects the positive technological spillover that firm i receives form R & D investments by others. The parameter $\delta \in [0, 1]$ is a constant depreciation rate that generates a permanent upward pressure on marginal costs. The cost of carrying out R & D is given by $\Gamma(k_i(t)) = b[k_i(t)]^2$, where b is a positive parameter. Firms discount future profits at a common and constant discount rate $\rho > 0$. The objective of firm i is therefore given by maximizing

$$\pi_i(k_i(t), q(t), c_i(t)) = \int_0^\infty \{[A - q_i(t) - Q_{-i}(t) - c_i(t)]q_i(t) - b[k_i(t)]^2\} e^{-\rho t} dt$$

subject to the relationship that determines the dynamics of marginal costs as a function of R & D efforts and initial conditions $c(0) = \{c_{0i}\}$.

The above model defines a non-cooperative game that is played by the firms, each firm choosing its output level and R & D expenditures.

Lemma 3.1 states that the open-loop Nash equilibrium of the game is subgame perfect. This result implies that the closed-loop equilibrium coincides with the open-loop one. As a consequence, observability of output levels and R & D expenditures by others is inconsequential for the strategic behavior of the firms. Proposition 3.1 gives closed form expressions for the steady-state point and states that this is the unique saddle-point equilibrium. Indeed, it is shown that provided $\delta\rho \leq A^2(1 + \beta(n-1))/[8b(n+1)]$, the steady state is given by

$$\begin{aligned} c^{ss} &= \frac{A(1+\beta(n-1)) - \sqrt{(1+\beta(n-1))[A^2(1+\beta(n-1)) - 8b\delta\rho(n+1)]}}{2(1+\beta(n-1))} \\ k^{ss} &= \frac{\delta}{[1+\beta(n-1)]}. \end{aligned}$$

Proposition 3.1 can be used to study the relationship between market structure (the number of firms n) and the investments in R & D. This relationship is central in the debate between Schumpeter and Arrow. The Schumpeterian view implies that monopoly leads to the highest R & D efforts, whereas the Arrowian position is that innovation is more attractive for a competitive firm. Proposition 4.1 states that the aggregate optimal R & D investment is monotonically increasing in the number of firms. This holds both along the equilibrium path and in the steady state. However, an increase in the number of firms causes a decrease in individual R & D effort, caused by two effects. First, tougher market competition reduces profits and therefore the funds available to any given firm for conducting R & D. Secondly, more firms means more positive externalities, so less individual incentives for R & D investment.