

M.R. GROSSINHO AND S. TERSIAN *An Introduction to Minimax Theorems and Their Applications to Differential Equations*. Dordrecht: Kluwer Academic Publishers, 2001. 269+xii p. (hc). (Nonconvex Optimization and Its Applications, 52). ISBN 0-7923-6832-0.

The book is an introduction to critical point theory and its applications to differential equations. The material has been organized in seven chapters.

Chapter I is an introductory chapter. It presents the variational principles of Ekeland and of Borwein-Preiss on the existence of  $\varepsilon$ -minimum points. It introduces a number of deformation lemmas, which are subsequently used to provide proofs of the mountain-pass theorem of Ambrosetti-Rabinowitz and some of its extensions. It also shows how mountain-pass theorems can be derived from the variational principle of Ekeland.

Chapter II starts with a discussion of the well-known minimax theorem of von Neumann. It also presents the saddle-point theorems of Rabinowitz and their extensions due to Lazer-Solimini on the one hand and Schechter on the other and shows how these results are connected to critical point theory. It formulates the critical point theorems of Brézis-Nirenberg and Li-Willem, and a number of linking theorems due to Silva.

Chapter III studies applications of critical point theorems to elliptic problems in bounded domains. The Neumann problem is studied, as well as the Hammerstein equations on a bounded domain with smooth boundary. The existence of solutions to the Hammerstein integral equations is shown using the Ambrosetti-Rabinowitz mountain-pass theorem discussed in Chapter I and the Lazer-Solimini saddle point theorem treated in Chapter II.

Chapter IV applies variational methods to prove the existence of periodic solutions of some second-order non-linear differential equations. Existence results are established using the Ambrosetti-Rabinowitz mountain-pass theorem of Chapter I and the Silva saddle-point the Li-Willem linking theorems of Chapter II.

The dual variational method is presented in Chapter V, as well as its applications to some problems for fourth-order differential equations with continuous and discontinuous nonlinearities.

The goal of Chapter VI is the presentation of a number of minimization and mountain-pass theorems for non-differentiable functionals. It is assumed that the functionals are locally Lipschitz, which implies that it is possible to define their generalized gradients. The mountain-pass theorems of Chang, Ghoussoub-Preiss and Brezis-Nirenberg are obtained as special cases of a general mountain-pass theorem.

Chapter VII concludes the book with an application of variational methods to study the existence of homoclinic and heteroclinic solutions of second-order equations, fourth-order equations and Schrödinger type equations.

The book is designed for graduate and postgraduate students as well as for specialists in

the fields of differential equations, variational methods and optimization. Although there is scope for improvement of the book, in particular with regard to the use of English and the exposition of the general intuition behind the results obtained, I have enjoyed reading the book for its mathematical clarity and rigor. I am convinced that any specialist in the field will use the book as a valuable reference manual, and I would therefore like to highly recommend it.

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