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D. Motreanu and P.D. Panagiotopoulos, *Minimax Theorems and Qualitative Properties of the Solutions of Hemivariational Inequalities.* Dordrecht: Kluwer Academic Publishers, 1999. 309+xviii p., prijs f 260,- (hc). (Nonconvex Optimization and Its Applications, 29). ISBN 0-7923-5456-7.

Boundary value problems which have variational expressions in the form of inequalities can be divided into problems leading to variational inequalities and problems leading to hemivariational inequalities. The latter class of problems is analysed in the book and result from applications in Mechanics and the Engineering Sciences, in particular applications related to nonconvex energy functions. The investigation of the qualitative properties of the solutions to such problems calls for a mathematical theory of eigenvalue problems for hemivariational inequalities.

The eigenvalue problems related to hemivariational inequalities have the following structure. Suppose that V is a Hilbert space, a is a bilinear form on $V \times V$, J is a locally Lipschitz functional on V and B is an operator mapping V into V^*, the dual space of V. When the duality pairing is denoted by $\langle \cdot, \cdot \rangle$, one eigenvalue problem consist of finding $\lambda \in \mathbb{R}$ and $u \in V$ to satisfy

$$a(u, v-u) + J^0(u, v-u) \geq \langle \lambda B u, v-u \rangle \quad \forall v \in V.$$

Here J^0 is the Clarke directional differential. These types of eigenvalue problems are generalizations of the classical eigenvalue problems for variational inequalities because J might be nonconvex.

Another eigenvalue problem consists of finding $\lambda \in \mathbb{R}$ and $u \in V$ to satisfy

$$a(u, v-u) + J^0(u, v-u) \geq (\lambda \overline{B} u, v-u) \quad \forall v \in V,$$

where $(\cdot, \cdot)$ denotes the inner product in V and $\overline{B}$ maps V into V.

The book consists of 9 chapters. The first three chapter summarize the general theory required to study these eigenvalue problems fruitfully. Chapter 1 presents a number of results in the field of nonsmooth analysis. Chapter 2 presents a number of general results on nonsmooth critical point theory that are applicable to locally Lipschitz functionals, and Chapter 3 the extension where the functionals are no longer locally Lipschitz.

Chapter 4 studies two types of eigenvalue problems for hemivariational inequalities. The chapter focuses on general existence results and applies the theory to a number of problems derived from mechanics. Multiplicity results for these eigenvalue problems are the topic of Chapter 5. That chapter develops a general minimax approach, which permits the use of linear eigenvalue problems for the determination of the eigenvectors of the hemivariational inequality.

Chapter 6 considers existence and multiplicity of solutions for a new type of eigenvalue problem for hemivariational inequalities whose solutions must belong to a sphere. This problem finds its origin in network flow problems. Chapter 7 studies a new type of eigenvalue problem that arises in the stability analysis of mechanical systems subject to nonmonotone boundary conditions. Chapter 8 studies existence, multiplicity and perturbation results for so-called "double eigenvalue problems."

The final chapter studies three problems: hyperbolic hemivariational inequalities, homoclinic solutions for second order Hamiltonian systems with discontinuous nonlinearities, and multiplicity results for hemivariational inequalities containing periodic energy functionals.

Although the book aims to be self-contained, it only partially succeeds in this ambition. For non-specialists in the field the book is rather inaccessible. The specialist on the other hand, will find a rich source of interesting and challenging eigenvalue problems, which will most certainly stimulate him or her in further research.