

Endogenously determined price rigidities^{*}

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Summary. There exists an extensive literature about economies with price rigidities, where some constraints on the set of admissible price systems are exogenously given. In this paper a general equilibrium model extended by a political system is described where the price rigidities are endogenously chosen by political candidates. Sufficient conditions for the existence of a mixed strategy and a pure strategy equilibrium are given. Finally an example is discussed, where in equilibrium both political candidates propose price rigidities excluding the Walrasian equilibrium price system.

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1 Introduction

Often government behaviour is considered to be exogenous in economic modelling. However, there exists no reason why government behaviour should not be explained, while behaviour of other agents acting in the economy is endogenously determined. The influence of the government on for example minimum wages is substantial and the existence of minimum wages clearly influences economic behaviour of producers and consumers. Moreover, the producers and consumers influence the level of the minimum wages by voting or by forming pressure groups. For example high levels of unemployment might increase the pressure on government to lower minimum wages. Hence, in order to explain the existence of minimum wages and to give an analysis of the most important determinants of the level of minimum wages, government behaviour should be modelled endogenously. The existence of minimum wages is just one example of a price rigidity. Other examples are price controls to reduce inflation (see Cox [8]), minimum prices for agricul-

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tural products, fixed exchange rates, price indexation, and the linkage between the wages of civil servants and the wages paid in industry. The existence of price regulations and price rigidities is a frequently occurring real world phenomenon. Nguyen and Whalley [28] (p. 667) make the same observation, stating: "Price controls have been employed by governments all over the world, during war and peace, in response to all manners of threats (both real and imaginary), and in all ages" and Levy [24] (p. 157) writes: "Price controls are pervasive in developing countries."

An important reason for the existence of price regulations is that they can be used to influence the redistribution of initial endowments. As Coughlin [4] argues, redistribution has become one of the most important political issues of the last three decades. A drawback of price regulations is the misallocation of resources resulting in efficiency losses.

In this paper a formal model capturing the ideas above is given. A stylized model of the political system corresponding with a democracy is described. The government consists of two political parties or candidates who compete for the votes of the consumers in the economy. For the sake of simplicity it is assumed that there are no producers in the economy. The political candidates have the possibility to propose price regulations in order to influence the redistribution of the initial endowments of the consumers.

The private sector will be modelled by a general equilibrium model. However, in case a political candidate proposes some regulation of the prices all the competitive equilibrium price systems might be excluded by the imposed price rigidities. Hence, an equilibrium in the classical sense cannot always be achieved. In order to obtain an equilibrium situation rationing may occur. The inclusion of price rigidities and rationing into general equilibrium models has been introduced by Bénassy [1], Drèze [12], and Younès [32]. In this paper the equilibrium concept of Drèze will be used in case price regulations are proposed.

In most of the existing literature about models with price rigidities some constraints on the set of admissible price systems are exogenously given. Some exceptions worth mentioning are Hart [17], Böhm, Maskin, Polemarchakis, and Postlewaite [2], Madden [25], and Silvestre [30]. In these papers (some of the) agents in the economy are price setters on some of the markets. In this way prices are endogenously determined and may be non-Walrasian and therefore possibly involve unemployment. However, it seems to be difficult to extend the results of these papers to general cases with more than three commodities.

In the model presented in this paper political candidates may impose price regulations on the markets. Both government behaviour and price rigidities are endogenously determined. The control of prices by government is also the subject of Nguyen and Whalley [27] and Ginsburgh and van der Heyden [14]. In these papers government behaviour and price rigidities are not endogenously determined, but the attention is focused on non-rationing mechanisms to solve the mismatch between supply and demand resulting from price regulations. This mismatch is solved by endogenously determined equilibrium

buying and selling prices in Nguyen and Whalley [27] and by government sales and purchases in Ginsburgh and van der Heyden [14].

Exchange economies with price rigidities will be treated in Section 2. The voting behaviour of consumers and the public sector are described in Section 3. In Section 4 sufficient conditions for the existence of a mixed strategy and a pure strategy Nash equilibrium for the game defined in Section 3 are given. In order to prove existence it is shown that for every economy there exists an upper bound, being chosen independently of the price regulations imposed, such that if the price on a market is above this upper bound then no trade takes place on this market. This result is quite intuitive and has some interest in itself. In Section 5 an example with two commodities and Cobb-Douglas utility functions is presented in which both political candidates impose price regulations excluding the Walrasian equilibrium price system, and therefore the resulting equilibrium is characterized by rationing.

2 Economies with price rigidities

In this paper political candidates are assumed to make proposals with respect to price regulations. In order to describe what happens in the economy if some particular price regulation is chosen by the political candidates, a model dealing with price regulations in an exchange economy is given in this section. Moreover, some important properties of this model are derived.

An economy with price regulations is denoted by $\mathcal{E}_{(p, \bar{p})} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(p, \bar{p})})$. In this economy there are m consumers, indexed $i = 1, \dots, m$, and $n + 1$ commodities, indexed $j = 0, \dots, n$. Consumer $i, i = 1, \dots, m$, is defined by a consumption set X^i , a preference relation \succeq^i on X^i , and a vector of initial endowments w^i . The vector of total initial endowments will be denoted by \bar{w} , so $\bar{w} = \sum_{i=1}^m w^i$. The set of admissible price systems is denoted by $P_{(p, \bar{p})}$. In this paper, $\{X^i, \succeq^i, w^i\}_{i=1}^m$ are always assumed to be given. The set of admissible price systems $P_{(p, \bar{p})}$ will be determined by the proposals of political candidates.

In the following, for $k \in \mathbb{N}$, I_k denotes the set of integers $\{1, \dots, k\}$ and I_k^0 denotes the set of integers $\{0, 1, \dots, k\}$. The following assumptions will be made with respect to an economy with price regulations $\mathcal{E}_{(p, \bar{p})}$.

- A1. For every consumer $i \in I_m$, the consumption set X^i is convex, closed, $X^i \subset \mathbb{R}_+^{n+1}$, and $X^i + \mathbb{R}_+^{n+1} \subset X^i$.
- A2. For every consumer $i \in I_m$, the preference relation \succeq^i on X^i is complete, transitive, continuous, strongly monotonic, and convex.
- A3. For every consumer $i \in I_m$, the initial endowments w^i are an element of the interior of X^i .
- A4. The set of admissible price systems is equal to $P_{(p, \bar{p})} = \{p \in \mathbb{R}_+^{n+1} \mid p_0 = 1, p_j \leq \bar{p}_j, \forall j \in I_n\}$, for some given $p \in \mathbb{R}_+^n$ and $\bar{p} \in \mathbb{R}_+^n$ with $p_j \leq \bar{p}_j$, for every $j \in I_n$.

If for a consumer $i \in I_m$, not $x^i \succeq^i y^i$ for two commodity bundles $x^i, y^i \in X^i$, then this will be denoted by $y^i \succ^i x^i$, and in case both $x^i \succeq^i y^i$ and $y^i \succeq^i x^i$ we denote this by $y^i \sim^i x^i$. The convexity assumption in A2 means that if $x^i, y^i \in X^i$ and if

$x^i \succ^i y^i$, then $\lambda x^i + (1 - \lambda)y^i \succ^i y^i, \forall \lambda \in (0, 1)$. The strong monotonicity assumption means that if $x^i, y^i \in X^i, x_j^i \leq y_j^i, \forall j \in I_n^0$, and $\exists j' \in I_n^0$ such that $x_{j'}^i < y_{j'}^i$, then $y^i \succ^i x^i$. As has been shown in Debreu [10], the Assumptions A1–A2 imply that it is possible to represent the preferences of a consumer $i \in I_m$ by a continuous utility function u^i from X^i into \mathbb{R} . In the following it is assumed that some particular representation u^i has been chosen.

The set of admissible price systems $P_{(p, \bar{p})}$ allows for a minimum price, p_j , and for a maximum price, \bar{p}_j , for commodity $j \in I_n$. Commodity 0 is a numeraire commodity with price equal to 1. For commodity $j \in I_n$ there may be total inflexibility of the price, i.e., $p_j = \bar{p}_j$, or a more moderate form of price rigidity, $p_j < \bar{p}_j$. Formally, on the market of each commodity some minimum and some maximum price is specified. However, in Theorem 2.4 it will be shown that if the minimum price of a commodity is taken small enough or the maximum price large enough, then this is equivalent to specifying no minimum price or no maximum price, respectively. Define the set R by $R = \{r \in \mathbb{R}_+^{2n} \mid r_j \leq r_{j+n}, \forall j \in I_n\}$. Notice that $P_{(p, \bar{p})}$ is a set of admissible price systems satisfying Assumption A4 if and only if $(p, \bar{p}) \in R$.

In the economy $\mathcal{E}_{(p, \bar{p})}$ a price system $p \in P_{(p, \bar{p})}$ at which for each commodity total supply and total demand are equal does not necessarily exist. In fact, this is the case if no Walrasian equilibrium price system is an element of $P_{(p, \bar{p})}$. Then the price mechanism is not capable of equating demand and supply on all markets.

Roughly spoken, when there is excess demand on one of the markets and the price of this commodity is equal to its upper bound, then the market will be cleared by quantity adjustments. Not all the demand can be satisfied, so the excess demand of some consumers has to be restricted in such a way that demand equals supply. When there is an excess demand on a market while the price has not yet reached the upper bound, an upward price adjustment will result. In such a case a quantity adjustment is not desired. When there is an excess supply on a market and the price of this commodity is equal to its lower bound, then the market can also be cleared by quantity adjustments. Again the excess supply of some consumers has to be restricted in such a way that demand equals supply. If there is an excess supply on a market while the price has not yet reached the lower bound, then no quantity adjustment will take place while a downward price adjustment will occur. These quantity adjustments will be called rationing in the following. Rationing should not affect excess supply and excess demand simultaneously to guarantee that markets are frictionless, and rationing should not force any consumer to exchange. Moreover, on the market of the numeraire commodity no rationing is allowed. These observations provide the motivation for the equilibrium concept of Drèze [12] for an economy with price regulations. It has to be remarked that alternative equilibrium concepts for economies with price rigidities have been provided by Bénassy [1] and Younès [32]. In Silvestre [29] conditions are given under which the approaches of Bénassy, Drèze, and Younès are equivalent. Moreover, it is shown that if no differentiability assumptions are made with respect to utility functions, then Drèze's definition is the strictest one. This

result and the mathematical convenience of Drèze's definition motivate the use of Drèze's equilibrium concept in this paper.

Before giving the definition of a Drèze equilibrium, some additional notation is needed. For the sake of simplicity it is assumed that there is uniform rationing, which means that the rationing scheme of every consumer is the same. The rationing scheme of every consumer is given by a pair of vectors $(l, L) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n$. The maximal net amount a consumer may supply of a commodity $j \in I_n$ is given by $-l_j$ and the maximal net amount a consumer may demand of commodity j is given by L_j . The budget set of a consumer $i \in I_m$ at rationing scheme $(l, L) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n$ and price system $p \in \mathbb{R}_+^{n+1}$ is given by the set

$$\gamma^i(l, L, p) = \{x^i \in X^i \mid p \cdot x^i \leq p \cdot w^i \text{ and } l_j \leq x_j^i - w_j^i \leq L_j, \forall j \in I_n\}.$$

The budget set of a consumer $i \in I_m$ is non-empty for all admissible l, L , and p , since $w^i \in \gamma^i(l, L, p)$. The demand of a consumer $i \in I_m$ at rationing scheme $(l, L) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n$ and price system $p \in \mathbb{R}_+^{n+1}$ is denoted by $\delta^i(l, L, p)$ and is given by

$$\delta^i(l, L, p) = \left\{ x^i \in \gamma^i(l, L, p) \mid u^i(x^i) = \max_{y^i \in \gamma^i(l, L, p)} u^i(y^i) \right\}.$$

Notice that the budget set and the demand of a consumer $i \in I_m$ are also defined for price systems p with $p_0 \neq 1$. This will be convenient in some of the subsequent proofs. If $p_0 > 0$ then the budget set $\gamma^i(l, L, p)$ is easily seen to be compact and therefore $\delta^i(l, L, p)$ is non-empty valued if $p_0 > 0$, even if some of the other prices $p_j, j \in I_n$, are equal to zero. Sometimes it will be useful in proofs to consider the following set. For every consumer $i \in I_m$, the correspondence $\Gamma^i: -\mathbb{R}_+^n \times \mathbb{R}_+^n \times \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+^{n+1}$ is defined by

$$\Gamma^i(l, L, p) = \{x^i \in \gamma^i(l, L, p) \mid x_0^i - w_0^i \leq \bar{w}_0\}, \forall (l, L, p) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n \times \mathbb{R}_+^{n+1}.$$

The corresponding demand correspondence of a consumer $i \in I_m, \Delta^i: -\mathbb{R}_+^n \times \mathbb{R}_+^n \times \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+^{n+1}$ has the advantage of being non-empty valued for every $(l, L, p) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n \times \mathbb{R}_+^{n+1}$.

Given $(l, L, p) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n \times \mathbb{R}_+^{n+1}$ consumer $i \in I_m$ is said to be constrained on his supply on market $k \in I_n$, or equivalently l_k is said to be binding for consumer $i \in I_m$, if $\tilde{x}^i \in \delta^i(\tilde{l}, L, p)$ and $x^i \in \delta^i(l, L, p)$ implies $\tilde{x}^i \succ^i x^i$, where \tilde{l} is the rationing scheme with $\tilde{l}_j = l_j, \forall j \in I_n \setminus \{k\}$, and $\tilde{l}_k = l_k - \varepsilon$ for some arbitrary positive real number ε . Using the convexity of preferences it is not difficult to show that if consumer $i \in I_m$ is constrained on market k then $x^i \in \delta^i(l, L, p)$ implies $l_k = x_k^i - w_k^i$, and if consumer $i \in I_m$ is not constrained on market k then $x^i \in \delta^i(l, L, p)$ implies $x^i \in \delta^i(\tilde{l}, L, p)$. Similar remarks can be made with respect to demand rationing. The remarks given above are also true for the correspondence Δ^i .

A Drèze equilibrium of an economy $\mathcal{E}_{(p, \bar{p})}$ is defined as follows (see Drèze [12]).

Definition 2.1 (Drèze Equilibrium). A Drèze equilibrium of the economy $\mathcal{E}_{(p,\bar{p})}$ is an element $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*)$ of the set $\prod_{i=1}^m X^i \times -\mathbb{R}_+^n \times \mathbb{R}_+^n \times P_{(p,\bar{p})}$ such that

1. $\forall i \in I_m: x^{*i} \in \delta^i(l^*, L^*, p^*);$
2. $\sum_{i=1}^m x^{*i} = \bar{w};$
3. $\forall j \in I_n: x_j^{*h} - w_j^h = L_j^*$ for some $h \in I_m$ implies $x_j^{*i} - w_j^i > l_j^* \forall i \in I_m$, and $x_j^{*h} - w_j^h = l_j^*$ for some $h \in I_m$ implies $x_j^{*i} - w_j^i < L_j^* \forall i \in I_m;$
4. $\forall j \in I_n: p_j^* < \bar{p}_j$ implies $L_j^* > x_j^{*i} - w_j^i \forall i \in I_m$, and $p_j^* > \underline{p}_j$ implies $l_j^* < x_j^{*i} - w_j^i \forall i \in I_m.$

Notice that the first two conditions are the standard ones, stating that every consumer behaves optimally given the price system and the rationing scheme, and that all markets clear. Condition 3 guarantees that markets are frictionless and Condition 4 makes sure that supply rationing can only occur if the price is on its lower bound, while demand rationing can only occur if the price is on its upper bound.

The set of Drèze equilibria of the economy $\mathcal{E}_{(p,\bar{p})}$ is denoted by $E_{(p,\bar{p})}^D$. In Drèze [12] it is shown that $E_{(p,\bar{p})}^D \neq \emptyset$ if the economy $\mathcal{E}_{(p,\bar{p})}$ satisfies the Assumptions A1–A4. Lemma 2.2 gives a result about the possible choices of the rationing schemes in a Drèze equilibrium. Lemma 2.2 shows that if $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(p,\bar{p})}^D$, then the rationing scheme (l^*, L^*) can be replaced by a rationing scheme (l, L) without changing the Drèze equilibrium allocation and price system, where (l, L) is chosen such that binding rationing schemes are not changed while non-binding rationing schemes may be changed in any way that keeps them non-binding. This lemma will be useful in showing that there is no loss of generality in considering only rationing schemes satisfying, for every $j \in I_n$, $l_j \geq -\bar{w}_j$ and $L_j \leq \bar{w}_j$.

Lemma 2.2. Let the economy $\mathcal{E}_{(p,\bar{p})}$ satisfy the Assumptions A1–A4 and let a Drèze equilibrium $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(p,\bar{p})}^D$ be given. Let $(l, L) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n$ be such that for every $j \in I_n$

- $$l_j^* = x_j^{*i} - w_j^i \text{ for some consumer } i \in I_m \text{ implies } l_j = l_j^*,$$
- $$l_j^* < x_j^{*i} - w_j^i \text{ for every consumer } i \in I_m \text{ implies } l_j < x_j^{*i} - w_j^i, \forall i \in I_m,$$
- $$L_j^* = x_j^{*i} - w_j^i \text{ for some consumer } i \in I_m \text{ implies } L_j = L_j^*, \text{ and}$$
- $$L_j^* > x_j^{*i} - w_j^i \text{ for every consumer } i \in I_m \text{ implies } L_j > x_j^{*i} - w_j^i, \forall i \in I_m.$$

Then $(x^{*1}, \dots, x^{*m}, l, L, p^*) \in E_{(p,\bar{p})}^D$.

Proof. Clearly, $(x^{*1}, \dots, x^{*m}, l, L, p^*)$ belongs to $\prod_{i=1}^m X^i \times -\mathbb{R}_+^n \times \mathbb{R}_+^n \times P_{(p,\bar{p})}$. It is easily seen that the Conditions 2, 3, and 4 of Definition 2.1 are satisfied by $(x^{*1}, \dots, x^{*m}, l, L, p^*)$, using the properties of l and L and the fact that $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(p,\bar{p})}^D$. So it remains to be shown that Condition 1 of a Drèze equilibrium is satisfied. Suppose for some $i \in I_m$, $x^{*i} \notin \delta^i(l, L, p^*)$. Clearly, $x^{*i} \in \gamma^i(l, L, p^*)$. Hence, for some $y^i \in \gamma^i(l, L, p^*)$, $y^i >^i x^{*i}$. Since $y^i \notin \gamma^i(l^*, L^*, p^*)$ and $y^i \in \gamma^i(l, L, p^*)$ it has to hold that for some $j \in I_n$, $l_j \leq y_j^i - w_j^i < l_j^*$, or for some $j \in I_n$, $L_j^* < y_j^i - w_j^i \leq L_j$. Moreover, it holds

that

$$[y_j^i - w_j^i < l_j^* \Rightarrow l_j^* < x_j^{*i} - w_j^i] \quad \text{and} \quad [y_j^i - w_j^i > L_j^* \Rightarrow L_j^* > x_j^{*i} - w_j^i]. \quad (1)$$

Let, for every $\lambda \in (0, 1)$, $y^i(\lambda)$ be defined by $y^i(\lambda) = \lambda y^i + (1 - \lambda)x^{*i}$. By the convexity of $\gamma^i(l, L, p^*)$ it holds that $y^i(\lambda) \in \gamma^i(l, L, p^*)$, $\forall \lambda \in (0, 1)$. By the convexity of \geq^i , $y^i(\lambda) \geq^i x^{*i}$, $\forall \lambda \in (0, 1)$. However, if λ is close enough to zero then $y^i(\lambda) \in \gamma^i(l^*, L^*, p^*)$, using (1) and the fact that $y^i(\lambda) \in \gamma^i(l, L, p^*)$. This is a contradiction since $x^{*i} \in \delta^i(l^*, L^*, p^*)$. Consequently, $(x^{*1}, \dots, x^{*m}, l, L, p^*)$ satisfies Condition 1 of a Drèze equilibrium. Q.E.D.

For every commodity both a lower and an upper bound were specified in the model described above. Theorem 2.4 will make clear that it is possible to use this model also in case on a market only a minimum price or a maximum price is given, or even in case there are no constraints on the prices at all, corresponding to the Walrasian case. Let $\underline{J} \subset I_n$ denote the possibly empty set of commodities on the market for which a minimum price is present, and let $\bar{J} \subset I_n$ denote the possibly empty set of commodities on the market for which a maximum price prevails. Again, for $j \in \underline{J}$, p_j denotes the minimum price on the market of commodity j and, for $j \in \bar{J}$, \bar{p}_j denotes the maximum price on the market of commodity j . Now it is possible to make the following assumption, which is weaker than Assumption A4.

A4'. The set of admissible price systems is equal to

$$P_{((p_j)_{j \in \underline{J}}, (\bar{p}_j)_{j \in \bar{J}})} = \{p \in \mathbb{R}_+^{n+1} \mid p_0 = 1, p_j \leq p_j, \forall j \in \underline{J}, p_j \leq \bar{p}_j, \forall j \in \bar{J}\},$$

with $p_j \in \mathbb{R}_+$, $\forall j \in \underline{J}$, $\bar{p}_j \in \mathbb{R}_+$, $\forall j \in \bar{J}$, and $p_j \leq \bar{p}_j$, $\forall j \in \underline{J} \cap \bar{J}$.

A Drèze equilibrium of an economy $\mathcal{E}_{((p_j)_{j \in \underline{J}}, (\bar{p}_j)_{j \in \bar{J}})} = (\{X^i, \geq^i, w^i\}_{i=1}^m, P_{((p_j)_{j \in \underline{J}}, (\bar{p}_j)_{j \in \bar{J}})})$ is defined according to Definition 2.1, where if $j \notin \bar{J}$ then $L_j^* > x_j^{*i} - w_j^i$, $\forall i \in I_m$, and if $j \notin \underline{J}$ then $l_j^* < x_j^{*i} - w_j^i$, $\forall i \in I_m$. So demand rationing is never allowed to be binding if no maximum price is specified and supply rationing is not binding if no minimum price is present. In case $\underline{J} = \bar{J} = \emptyset$ one obtains the definition of a Walrasian equilibrium. The set of Drèze equilibria of the economy $\mathcal{E}_{((p_j)_{j \in \underline{J}}, (\bar{p}_j)_{j \in \bar{J}})}$ is again denoted by $E_{((p_j)_{j \in \underline{J}}, (\bar{p}_j)_{j \in \bar{J}})}^D$.

Before showing Theorem 2.4 a lemma is needed that gives some properties of the correspondence Δ^i , $i \in I_m$. For every $M \in \mathbb{R}_+$ a set of rationing schemes and price systems, denoted by \mathcal{L}_M , is defined by

$$\begin{aligned} \mathcal{L}_M = \{ & (l, L, p) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n \times \{1\} \times \mathbb{R}_+^n \mid \forall j \in I_n, l_j \geq -\bar{w}_j \text{ and } L_j \leq \bar{w}_j, \\ & \exists k \in I_n \text{ such that } p_k \geq M \text{ and } l_k = -\bar{w}_k, \text{ and} \\ & \text{if } l_j < 0 \text{ for some } j \in I_n \text{ then there exists } k \in I_n \text{ such that } p_k \geq p_j \text{ and} \\ & l_k = -\bar{w}_k \}. \end{aligned}$$

If $(l, L, p) \in \mathcal{L}_M$ then there is at least one commodity $k \in I_n$ with price greater than or equal to M and with $l_k = -\bar{w}_k$ which guarantees that no consumer is constrained on his supply on market k . Moreover, if it is possible to supply a positive amount of some commodity $j \in I_n$, i.e., $l_j < 0$, then there exists a commodity $k \in I_n$ with price at least as high as p_j and with $l_k = -\bar{w}_k$. Finally,

the rationing schemes in \mathcal{L}_M are bounded, for every commodity $j \in I_n$ it holds that $-\bar{w}_j \leq l_j \leq 0$ and $0 \leq L_j \leq \bar{w}_j$. It should be noticed that $M^1 > M^2$ implies $\mathcal{L}_{M^1} \subset \mathcal{L}_{M^2}$.

Lemma 2.3 states that if M is sufficiently large then for every consumer $i \in I_m$ the demand of commodity 0 according to Δ^i at price systems and rationing schemes in the set \mathcal{L}_M exceeds the total initial endowments of commodity 0. The intuition behind this result is the following. There exists a commodity $k \in I_n$ with price at least equal to M and with $l_k = -\bar{w}_k$, implying that there are supply possibilities for this commodity. If M is high enough, then the value of the numeraire commodity is relatively very low, implying that it is very attractive to demand the numeraire commodity in exchange for a commodity having a relatively high price and for which there are supply possibilities. Moreover, by taking M sufficiently high, it is even possible to guarantee that the demand of the numeraire commodity of any consumer exceeds the total initial endowments of the numeraire commodity. Lemma 2.3 is closely related to theorems providing boundary conditions on demand functions without taking into account the possibility of rationing, see for example Proposition 3.1.4 of Hildenbrand and Kirman [18]. Lemma 2.3 can therefore be considered as an extension of those theorems to the case where rationing is allowed.

Lemma 2.3. *Let $\{X^i, \succeq^i, w^i\}_{i=1}^m$ satisfy the Assumptions A1–A3. Then there exists $M \in \mathbb{R}_+$ such that if $(l, L, p) \in \mathcal{L}_M$ then, for every $i \in I_m$, $x^i \in \Delta^i(l, L, p)$ implies $x_0^i > \bar{w}_0$.*

Proof. Suppose Lemma 2.3 is false. Then there is $i \in I_m$ and a sequence $(l^r, L^r, p^r, x^{ir})_{r \in \mathbb{N}}$ in $\mathcal{L}_r \times X^i$ such that $x^{ir} \in \Delta^i(l^r, L^r, p^r)$ and $x_0^{ir} \leq \bar{w}_0$. Consider the bounded sequence

$$\left(l^r, L^r, \frac{p^r}{\|p^r\|}, x^{ir} \right)_{r \in \mathbb{N}}, \tag{2}$$

which without loss of generality can be assumed to converge to an element $(l, L, 0, p', x^i) \in -\mathbb{R}_+^n \times \mathbb{R}_+^n \times \{0\} \times \mathbb{R}_+^n \times X^i$. Notice that since $x^{ir} \in \Gamma^i(l^r, L^r, p^r)$ and $(l^r, L^r, p^r) \in \mathcal{L}_r$, x^{ir} remains indeed in a bounded set. Moreover, $(l^r, L^r, p^r) \in \mathcal{L}_r$ implies that $\|p^r\| \rightarrow \infty$ if $r \rightarrow \infty$. By the homogeneity of degree zero in prices of the demand correspondence it holds that, for every $r \in \mathbb{N}$, $x^{ir} \in \Delta^i\left(l^r, L^r, \frac{p^r}{\|p^r\|}\right)$.

Suppose $p' \cdot l > 0$, then by the lemma in Drèze [12] (p. 304) it holds that Δ^i is upper semi-continuous at $(l, L, 0, p')$ and therefore $x^i \in \Delta^i(l, L, 0, p')$. By the strict monotonicity of preferences it holds that $x_0^i - w_0^i = \bar{w}_0$. This contradicts $x_0^{ir} \leq \bar{w}_0, \forall r \in \mathbb{N}$. Consequently, $p' \cdot l = 0$.

Suppose there exists a subsequence $\left(l^{rs}, L^{rs}, \frac{p^{rs}}{\|p^{rs}\|}, x^{irs} \right)_{s \in \mathbb{N}}$ of the sequence in (2) such that for some $k \in I_n$ with $p'_k > 0$, for every $s \in \mathbb{N}$, l_k^{rs} is not binding for consumer i . Then $x^{irs} \in \Delta^i(\tilde{l}^{rs}, L^{rs}, p^{rs})$, where \tilde{l}^{rs} is the rationing scheme with $\tilde{l}_j^{rs} = l_j^{rs}$,

$\forall j \in I_n \setminus \{k\}$, and $\tilde{l}_k^r = -\bar{w}_k$. By considering the sequence $\left(\tilde{l}^r, L^r, \frac{p^r}{\|p^r\|}, x^{r^s}\right)_{s \in \mathbb{N}}$ one obtains a contradiction in the same way as in the previous paragraph. Consequently, $p' \cdot l = 0$ and, without loss of generality, for every $r \in \mathbb{N}$, for every $k \in I_n$ with $p'_k > 0$, $l'_k = x_k^r - w_k^i$.

Suppose there exists a subsequence $\left(l^r, L^r, \frac{p^r}{\|p^r\|}, x^{r^s}\right)_{s \in \mathbb{N}}$ of the sequence in (2) such that for some $k \in I_n$ with $p'_k > 0$, for every $s \in \mathbb{N}$, $l_{k^s}^r < 0$. Since $(l^r, L^r, p^r) \in \mathcal{L}_r$, it holds for some $k^r \in I_n$ that $p_{k^r}^r \geq p_k^r$ and $l_{k^r}^r = -\bar{w}_{k^r}$. This contradicts $p' \cdot l = 0$. Consequently, $p' \cdot l = 0$ and, without loss of generality, for every $r \in \mathbb{N}$, for every $k \in I_n$ with $p'_k > 0$, $0 = l'_k = x_k^r - w_k^i$.

Since there exists $j \in I_n$ such that $l_j = -\bar{w}_j$ and since $p' \cdot l = 0$, the set J , defined by $J = \{k \in I_n \mid p'_k > 0\}$, is a non-empty proper subset of I_n . Without loss of generality $J = I_n \setminus I_{\hat{n}}$ for some $\hat{n} \in \mathbb{N}$ satisfying $1 \leq \hat{n} < n$. Now consider a consumer with characteristics $\hat{X}^i, \hat{\Sigma}^i, \hat{w}^i$, where $\hat{X}^i = \{x^i \in \mathbb{R}^{\hat{n}+1} \mid (x^i, w_{\hat{n}+1}^i, \dots, w_n^i) \in X^i\}$, $\hat{\Sigma}^i = \{(x^i, y^i) \in \hat{X}^i \times \hat{X}^i \mid (x^i, w_{\hat{n}+1}^i, \dots, w_n^i) \geq^i (y^i, w_{\hat{n}+1}^i, \dots, w_n^i)\}$, $\hat{w}^i = (w_0^i, \dots, w_{\hat{n}}^i)$. It is easily verified that $(\hat{X}^i, \hat{\Sigma}^i, \hat{w}^i)$ satisfies the Assumptions A1–A3. Moreover, using that $x_k^r = w_k^i, \forall k \in J$, it is clear that the corresponding demand correspondence $\hat{\Delta}^i$ satisfies $\hat{x}^{r'} \in \hat{\Delta}^i(\hat{l}^r, \hat{L}^r, \hat{p}^r), \forall r \in \mathbb{N}$. Here \hat{l}^r and \hat{L}^r denote the first \hat{n} components of l^r and L^r , respectively, and \hat{p}^r and $\hat{x}^{r'}$ denote the first $\hat{n} + 1$ components of p^r and $x^{r'}$, respectively. Since, for every $j \in I_n \setminus I_{\hat{n}}, l_j = 0$, the definition of \mathcal{L}_r implies that there is no loss of generality in assuming that for every $r \in \mathbb{N}$ there exists $k^r \in I_{\hat{n}}$ such that $\hat{p}_{k^r}^r \geq r$ and $\hat{l}_{k^r}^r = -\bar{w}_{k^r}$. Consider the bounded sequence $\left(\hat{l}^r, \hat{L}^r, \frac{\hat{p}^r}{\|\hat{p}^r\|}, \hat{x}^{r'}\right)_{r \in \mathbb{N}}$ in $-\mathbb{R}_+^{\hat{n}} \times \mathbb{R}_+^{\hat{n}} \times \mathbb{R}_+^{\hat{n}+1} \times \hat{X}^i$. Without loss of generality this sequence converges to an element $(\hat{l}, \hat{L}, 0, \hat{x}') \in -\mathbb{R}_+^{\hat{n}} \times \mathbb{R}_+^{\hat{n}} \times \{0\} \times \mathbb{R}_+^{\hat{n}} \times \hat{X}^i$. Suppose $\hat{p}' \cdot \hat{l} > 0$, then again by Drèze [12] $\hat{\Delta}^i$ is upper semi-continuous at $(\hat{l}, \hat{L}, 0, \hat{p}')$ and therefore $\hat{x}' \in \hat{\Delta}^i(\hat{l}, \hat{L}, 0, \hat{p}')$, yielding a contradiction in the same way as before. Repeating the arguments used before, the finiteness of n and the definition of \mathcal{L}_r guarantees that in a finite number of steps the case where $\hat{p}' \cdot \hat{l} > 0$ will be reached, contradicting that for every $r \in \mathbb{N}, \hat{x}_0^r \leq \bar{w}_0$. Q.E.D

Using Lemma 2.3 it can be shown that specifying no lower bound or specifying no upper bound on a price is equivalent to specifying a lower bound being equal to zero or a sufficiently high upper bound on a price, respectively. Clearly, no equilibria are lost by specifying a lower bound being equal to zero. Moreover, no equilibria are lost by specifying a sufficiently high upper bound, since Lemma 2.3 can be used to show that if the price on a market becomes sufficiently high, then every consumer has a positive excess demand of the numeraire commodity, being incompatible with a Drèze equilibrium. It is easily shown that no new equilibria are generated by specifying the lower and upper bounds as described above.

Theorem 2.4. *Let the economy $\mathcal{E}_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}$ satisfy the Assumptions A1–A3 and A4'. Then there exists $(\underline{p}', \bar{p}') \in R$ such that $E_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}^D = E_{(\underline{p}', \bar{p}')}^D$.*

Proof. Let M be as in Lemma 2.3 and take $N \in \mathbb{R}$ such that $N \geq M$ and $N > p_j, \forall j \in J$. For $j \in \underline{J}$ let $p'_j = p_j$, for $j \in I_n \setminus \underline{J}$ let $p'_j = 0$, for $j \in \bar{J}$ let $\bar{p}'_j = \bar{p}_j$, and for $j \in I_n \setminus \bar{J}$ let $\bar{p}'_j = N$. This defines the economy $\mathcal{E}_{(\underline{p}', \bar{p}'')}$.

Suppose $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}^D$, whereas $\|p^*\|_\infty > N$. Using the same arguments as in Lemma 2.2 and the fact that $\sum_{i=1}^m x^{*i} = \bar{w}$, there is no loss of generality in assuming that $l_j^* = -\bar{w}_j$ if $j \in I_n$ satisfies $l_j^* < x_j^{*i} - w_j^i, \forall i \in I_m$, and that $L_j^* = \bar{w}_j$ if $j \in I_n$ satisfies $L_j^* > x_j^{*i} - w_j^i, \forall i \in I_m$. Since $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}^D$, it then holds that $l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j, \forall j \in I_n$. For all $h \in I_m, x^{*h} \in \delta^h(l^*, L^*, p^*)$, and since $x_0^{*h} - w_0^h \leq \sum_{i=1}^m x_0^{*i} = \bar{w}_0$ it holds that $x^{*h} \in \Delta^h(l^*, L^*, p^*)$. For every $k \in I_n$ with $p_k^* \geq N$ it follows that $l_k^* = -\bar{w}_k$ since $N > p_j, \forall j \in \underline{J}$. So $(l^*, L^*, p^*) \in \mathcal{L}_M$ and therefore $x_0^{*i} > \bar{w}_0, \forall i \in I_m$, contradicting Condition 2 of Definition 2.1. Consequently, for every $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}^D, \|p^*\|_\infty \leq N$. Using the definition of a Drèze equilibrium it follows now immediately that $E_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}^D \subset E_{(\underline{p}', \bar{p}'')}^D$. Let $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(\underline{p}', \bar{p}'')}^D$ be given. Using Lemma 2.2 and the fact that $\sum_{i=1}^m x^{*i} = \bar{w}$ there is no loss of generality in assuming that $l_j^* = -\bar{w}_j$ if $j \in I_n$ satisfies $l_j^* < x_j^{*i} - w_j^i, \forall i \in I_m$, and that $L_j^* = \bar{w}_j$ if $j \in I_n$ satisfies $L_j^* > x_j^{*i} - w_j^i, \forall i \in I_m$. Since $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(\underline{p}', \bar{p}'')}^D$ it then holds that $l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j, \forall j \in I_n$. Suppose $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \notin E_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}^D$. Then for some $j \in I_n \setminus \underline{J}, p_j^* = 0$ and $l_j^* = x_j^{*i} - w_j^i$ for some $i \in I_m$, or for some $j \in I_n \setminus \bar{J}, p_j^* = N$ and $L_j^* = x_j^{*i} - w_j^i$ for some $i \in I_m$. Consider the first case. If $p_j^* = 0$ then by the strict monotonicity of the preference relation it holds that $x_j^{*i} - w_j^i = L_j^* > l_j^* = x_j^{*i} - w_j^i$, where for the inequality Condition 3 of Definition 2.1 is used. This is a contradiction. Consider the second case. By Condition 4 of a Drèze equilibrium for every $j \in I_n$ for which $p_j^* = N$ it holds that $l_j^* = -\bar{w}_j$, so $(l^*, L^*, p^*) \in \mathcal{L}_M$. Clearly, for every $i \in I_m, x^{*i} \in \Delta^i(l^*, L^*, p^*)$ and hence $x_0^{*i} > \bar{w}_0$, contradicting Condition 2 of Definition 2.1. Consequently, $E_{(\underline{p}', \bar{p}'')}^D \subset E_{((p_j)_{j \in J}, (\bar{p}_j)_{j \in \bar{J}})}^D$. Q.E.D.

Let an economy $\mathcal{E}_{(\underline{p}, \bar{p})}$ be given. In Theorem 2.5 it is shown that it is possible to represent the admissible price systems and rationing schemes by the set

$$Q = \{q \in \mathbb{R}^n \mid 0 \leq q_j \leq 1, \forall j \in I_n\},$$

of which an element q will be called a pseudo-price vector. For every $j \in I_n$, the j -th component of the functions $\hat{p}: R \times Q \rightarrow \{1\} \times \mathbb{R}_+^n, \hat{l}: Q \rightarrow -\mathbb{R}_+^n$, and $\hat{L}: Q \rightarrow \mathbb{R}_+^n$ is defined by

$$\begin{aligned} \hat{p}_j(\underline{p}, \bar{p}, q) &= \max\{p_j, \min\{\bar{p}_j, p_j(2 - 3q_j) + \bar{p}_j(3q_j - 1)\}\}, \forall (\underline{p}, \bar{p}) \in R, \forall q \in Q, \\ \hat{l}_j(q) &= -\min\{1, 3q_j\}\bar{w}_j, \forall q \in Q, \\ \hat{L}_j(q) &= \min\{1, 3 - 3q_j\}\bar{w}_j, \forall q \in Q. \end{aligned}$$

Notice that $\hat{p}_0(\underline{p}, \bar{p}, q) = 1, \forall (\underline{p}, \bar{p}) \in R, \forall q \in Q$. In this way a pseudo-price $q \in Q$ determines an admissible price system $\hat{p}(\underline{p}, \bar{p}, q)$ and a rationing scheme

$(\hat{l}(q), \hat{L}(q))$. Remark that for every $j \in I_n$,

$$\begin{aligned} 0 &< q_j < \frac{1}{3} \text{ implies } \hat{p}_j(\underline{p}, \bar{p}, q) = \underline{p}_j, \hat{l}_j(q) > -\bar{w}_j, \text{ and } \hat{L}_j(q) = \bar{w}_j, \\ \frac{1}{3} &< q_j \leq \frac{2}{3} \text{ implies } \underline{p}_j \leq \hat{p}_j(\underline{p}, \bar{p}, q) \leq \bar{p}_j, \hat{l}_j(q) = -\bar{w}_j, \text{ and } \hat{L}_j(q) = \bar{w}_j, \\ \frac{2}{3} &< q_j \leq 1 \text{ implies } \hat{p}_j(\underline{p}, \bar{p}, q) = \bar{p}_j, \hat{l}_j(q) = -\bar{w}_j, \text{ and } \hat{L}_j(q) < \bar{w}_j. \end{aligned}$$

These properties will guarantee that Conditions 3 and 4 of Definition 2.1 are satisfied if price systems and rationing schemes are described using the pseudo-prices. Clearly, the choice of the values $\frac{1}{3}$ and $\frac{2}{3}$ to induce switches of regimes is an arbitrary choice. In the following theorem it is shown that describing price systems and rationing schemes by pseudo-prices does not exclude any of the Drèze equilibrium allocations and price systems.

Theorem 2.5. *Let the economy $\mathcal{E}_{(p, \bar{p})}$ satisfy the Assumptions A1–A4. Let a Drèze equilibrium $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(p, \bar{p})}^D$ be given. Then there exists a pseudo-price $q^* \in Q$ such that $(x^{*1}, \dots, x^{*m}, \hat{l}(q^*), \hat{L}(q^*), \hat{p}(\underline{p}, \bar{p}, q^*)) \in E_{(p, \bar{p})}^D$, while $\hat{p}(\underline{p}, \bar{p}, q^*) = p^*$.*

Proof. Let the sets J^1, J^2 , and J^3 be defined by

$$\begin{aligned} J^1 &= \{j \in I_n \mid \exists i \in I_m: x_j^{*i} - w_j^i = L_j^*\}, \\ J^2 &= \{j \in I_n \mid \forall i \in I_m: l_j^* < x_j^{*i} - w_j^i < L_j^*\}, \\ J^3 &= \{j \in I_n \mid \exists i \in I_m: l_j^* = x_j^{*i} - w_j^i\}. \end{aligned}$$

By condition 3 of Definition 2.1, for every $j \in J^1, l_j^* < x_j^{*i} - w_j^i, \forall i \in I_m$, and for every $j \in J^3, x_j^{*i} - w_j^i < L_j^*, \forall i \in I_m$. By Condition 4 of Definition 2.1, for every $j \in J^1, p_j^* = \bar{p}_j$, and, for every $j \in J^3, p_j^* = \underline{p}_j$. Obviously, $\{J^1, J^2, J^3\}$ is a partition of I_n . Clearly, for every $j \in I_n, -\bar{w}_j < x_j^{*i} - w_j^i < \bar{w}_j, \forall i \in I_m$. Let $l \in \mathbb{R}_+^n$ be defined by $l_j = -\bar{w}_j, \forall j \in J^1 \cup J^2$, and $l_j = l_j^*, \forall j \in J^3$. Let $L \in \mathbb{R}_+^n$ be defined by $L_j = \bar{w}_j, \forall j \in J^2 \cup J^3$, and $L_j = L_j^*, \forall j \in J^1$. Then by Lemma 2.2 it holds that $(x^{*1}, \dots, x^{*m}, l, L, p^*) \in E_{(p, \bar{p})}^D$. Moreover, for every $j \in I_n, l_j \geq -\bar{w}_j$ and $L_j \leq \bar{w}_j$. Let $q^* \in Q$ be defined by

$$\begin{aligned} q_j^* &= 1 - \frac{L_j}{3\bar{w}_j}, \forall j \in J^1, \\ q_j^* &= \frac{1}{2}, \forall j \in J^2 \text{ such that } p_j = \bar{p}_j, \\ q_j^* &= \frac{p_j^* + \bar{p}_j - 2\underline{p}_j}{3(\bar{p}_j - \underline{p}_j)}, \forall j \in J^2 \text{ such that } \underline{p}_j < \bar{p}_j, \\ q_j^* &= -\frac{l_j}{3\bar{w}_j}, \forall j \in J^3. \end{aligned}$$

It is easily checked that $\hat{l}(q^*) = l, \hat{L}(q^*) = L, \hat{p}(\underline{p}, \bar{p}, q^*) = p^*$, and therefore $(x^{*1}, \dots, x^{*m}, \hat{l}(q^*), \hat{L}(q^*), \hat{p}(\underline{p}, \bar{p}, q^*)) \in E_{(p, \bar{p})}^D$. Q.E.D.

3 Endogenously determined price rigidities

The behaviour of the government will be modelled as being the result of the competition for votes between two political candidates, indexed by $k = 1, 2$. It

is not difficult to extend the model in a similar way as in Wittman [31] and allow for an arbitrary number of political candidates. The electorate consists of the consumers in the economy and chooses between the political candidates by a majority vote.

Political candidates are assumed to have the possibility to propose price regulations on the markets. For $k \in I_2$, $A^k \subset R$ denotes the set of admissible price regulations among which political candidate k can choose. An element a^k of A^k of the political candidates corresponds with the choice of a lower bound and an upper bound on the set of admissible price systems. If political candidate $k \in I_2$ chooses a policy position $a^k \in A^k$ then the proposed set P_{a^k} of admissible price systems of the economy is given by

$$P_{a^k} = \{p \in \mathbb{R}_+^{n+1} \mid p_0 = 1, a_j^k \leq p_j \leq a_{j+n}^k, \forall j \in I_n\}$$

and the resulting economy is given by $\mathcal{E}_{a^k} = (\{X^i, \geq^i, w^i\}_{i=1}^m, P_{a^k})$. In general more than one Drèze equilibrium allocation and price system might result for the economy, given any lower bound and any upper bound on the prices. For instance the results of Debreu [11] imply that economies exist with an arbitrary number of Walrasian equilibria. It will be assumed that a political candidate proposes besides the chosen price regulation also a corresponding Drèze equilibrium price system, an amount of supply rationing (also called unemployment), and an amount of demand rationing on each market. By Theorem 2.5 there is no loss of generality in describing price systems and rationing schemes by the pseudo-price $q \in Q$. The set C^k will denote the set of admissible actions of political candidate $k \in I_2$ and is therefore given by

$$C^k = \{(a^k, q) \in A^k \times Q \mid \text{there exists a Drèze equilibrium } (x^1, \dots, x^m, \hat{l}(q), \hat{L}(q), \hat{p}(a^k, q)) \text{ of } \mathcal{E}_{a^k}\}. \tag{3}$$

When $A^k = R$ then the corresponding set of admissible actions C^k is denoted by C . The indirect utility function $\bar{v}^i: C \rightarrow \mathbb{R}$ of a consumer $i \in I_m$ is defined by associating with every $(a, q) \in C$ the real number $\bar{v}^i(a, q)$ satisfying

$$\bar{v}^i(a, q) = u^i(x^{*i}), \text{ for } x^{*i} \in \delta^i(\hat{l}(q), \hat{L}(q), \hat{p}(a, q)).$$

Obviously, this function is well defined.

In order to describe the assumptions with respect to the set of admissible price regulations of the political candidates, a new mathematical concept will be introduced first. Define for $v \in \mathbb{N}$ and $r \in \mathbb{R}_+$ the v -dimensional cube $B_r^v = \{b \in \mathbb{R}^v \mid \|b\|_\infty \leq r\}$ and define the projection function $\beta_r^v: \mathbb{R}^v \rightarrow B_r^v$ which assigns to an element $x \in \mathbb{R}^v$ the element $b \in B_r^v$ minimizing $\|x - b\|_2$, so for every $j \in I_v$, $(\beta_r^v(x))_j = \min\{\max\{-r, x_j\}, r\}$. Clearly, β_r^v is a continuous function. Now it is possible to give the definition of a property of a subset of a Euclidean space, weaker than compactness but stronger than closedness, and hence called semi-compactness.

Definition 3.1 (Semi-Compactness). *A subset S of \mathbb{R}^v is semi-compact if $\beta_r^v(S)$ is closed for every $r \in \mathbb{R}_+$.*

Since the continuous image of a compact set is compact, every compact subset of \mathbb{R}^v is semi-compact. It will be shown that every semi-compact set is closed. Suppose $S \subset \mathbb{R}^v$ is semi-compact but not closed. Then there exists a converging sequence of points in S with limit s such that $s \notin S$. Let $r \in \mathbb{R}_+$ be such that $r > \|s\|_\infty$. Then $\beta_r^v(S)$ is not closed, which contradicts the semi-compactness of S . Two examples of semi-compact sets not being compact are the sets \mathbb{R}^v and \mathbb{N}^v . An example of a closed set not being semi-compact is the set $\{s \in \mathbb{R}^2 \mid s_2 = \arctan(s_1)\}$. It is not difficult to show that for subsets of \mathbb{R} the concepts of semi-compactness and closedness coincide. When verifying the semi-compactness of a set it is useful to know that if $0 \leq r \leq \bar{r}$ and $\beta_r^v(S)$ is closed then $\beta_{\bar{r}}^v(S)$ is closed. This follows easily from the property that $\beta_r^v(\beta_{\bar{r}}^v(S)) = \beta_{\bar{r}}^v(S)$ for $r \leq \bar{r}$. If $\beta_{\bar{r}}^v(S)$ is closed then it has to be compact, and since the continuous image of a compact set is compact and therefore closed, the set $\beta_r^v(\beta_{\bar{r}}^v(S))$ is closed for any $r \leq \bar{r}$. The following assumption with respect to the set A^k is made.

B1. For every political candidate $k \in I_2$, the set of admissible price regulations $A^k \subset R$ is non-empty and semi-compact.

The assumptions made with respect to A^k , $k \in I_2$, are very weak. Since $\beta_r^{2n}(R) = B_r^{2n} \cap R$, which is an intersection of two closed sets and therefore closed for every $r \geq 0$, the set R itself satisfies Assumption B1. So the case $A^k = R$, $k \in I_2$, is not excluded. This is conceptually the most interesting case, since it corresponds to the situation where in a democratic society price regulations are chosen by political candidates, and where there are no restrictions on the set of admissible price regulations.

However, it is also possible to model that a political candidate is not capable of setting arbitrarily chosen lower and upper bounds on the prices, for example because of institutional reasons. This might be the more realistic case, since according to Cox [8] regulators are not capable of enforcing every possible price regulation. This might be modelled by restricting the set A^k of admissible price regulations to be some non-empty compact subset of R . Another possibility is that each political candidate is only capable of considering a finite number of possibilities, in which case the set A^k is a finite set. Assumption B1 also admits many intermediate possibilities for the set A^k , for example cases where political candidates are only able to regulate prices on some markets. An example for $n=2$ is given by the semi-compact set $A^k = \{(p_1, p_2, \bar{p}_1, \bar{p}_2) \in R \mid p_2 = \bar{p}_2 = 1\}$.

According to Kramer [23] deterministic voting equilibria only exist under extremely restrictive assumptions in case the policy space is more than one-dimensional, which is clearly the case in this paper. This is why attention will also be focused on probabilistic voting models, where political candidates do not necessarily have perfect information about the voting decision of consumers. Voting models with some probabilistic aspects were first rigorously analyzed in Hinich and Ordeshook [21], [22], and Hinich, Ledyard and Ordeshook [20]. For the sake of simplicity in this paper probabilistic voting models without abstentions will be considered following the approach of

among others, Comanor [3], Coughlin and Nitzan [7], and Feldman and Lee [13]. In Wittman [31] the following two arguments for the probabilistic voting model are given. The first argument is that political candidates do not have perfect information about the preferences and actions of the voters. This is also the point of view taken in Coughlin, Mueller and Murrell [5] where in the preferences of the voters there is a bias in favour of or against a political candidate not perfectly known to the political candidates. The second argument is that voters do not have perfect information about the positions of the political candidates when casting their vote.

It will be assumed that political candidates have the same subjective expectations about the voting behaviour of the consumers. This assumption can easily be relaxed, but is made for notational convenience. For political candidate $k \in I_2$ and for every consumer $i \in I_m$ a voting function $\pi^{ki}: \bar{v}^i(C^1) \times \bar{v}^i(C^2) \rightarrow [0, 1]$ describes the expectations of a political candidate about the voting behaviour of consumer i . If the political candidates have chosen actions $c^1 \in C^1$ and $c^2 \in C^2$ then $\pi^{ki}(\bar{v}^i(c^1), \bar{v}^i(c^2))$ is the probability a political candidate assigns to the event that a consumer $i \in I_m$ votes for a political candidate $k \in I_2$ if the proposal of political candidate 1 yields consumer i a utility level $\bar{v}^i(c^1)$ and the proposal of political candidate 2 yields consumer i a utility level $\bar{v}^i(c^2)$. Notice that voting without abstentions implies that $\pi^{1i}(v^{1i}(v^{2i})) + \pi^{2i}(v^{1i}, v^{2i}) = 1$, $\forall (v^{1i}, v^{2i}) \in \bar{v}^i(C^1) \times \bar{v}^i(C^2)$. This completes the description of the political economic system $\mathcal{E} = (\{X^i, \succeq^i, w^i\}_{i=1}^m, \{A^k, \{\pi^{ki}\}_{i=1}^m\}_{k=1}^2)$. Deterministic voting without abstentions corresponds to the case where $\pi^{1i}(v^{1i}, v^{2i}) = 1$ if $v^{1i} > v^{2i}$, $\pi^{1i}(v^{1i}, v^{2i}) = \frac{1}{2}$ if $v^{1i} = v^{2i}$, $\pi^{1i}(v^{1i}, v^{2i}) = 0$ if $v^{1i} < v^{2i}$, and $\pi^{2i}(v^{1i}, v^{2i}) = 1 - \pi^{1i}(v^{1i}, v^{2i})$, $\forall (v^{1i}, v^{2i}) \in \bar{v}^i(C^1) \times \bar{v}^i(C^2)$. As a matter of realism the assumption that π^{ki} is non-decreasing in v^{ki} and non-increasing in $v^{k'i}$, $k' \neq k$, is often made. For this paper the only assumption needed with respect to the voting functions is the following.

B2. For every consumer $i \in I_m$, for every political candidate $k \in I_2$, the function $\pi^{ki}: \bar{v}^i(C^1) \times \bar{v}^i(C^2) \rightarrow [0, 1]$ is continuous.

In case both $\bar{v}^i(C^1)$ and $\bar{v}^i(C^2)$ have no accumulation points Assumption B2 does not exclude any function π^{ki} and therefore does not exclude deterministic voting behaviour. This is for example the case if the sets C^1 and C^2 are finite.

The preference relation \succeq^i on X^i of a consumer $i \in I_m$ admits many representations by utility functions. Now consider the case where such a representation \hat{u}^i (not necessarily continuous) different from u^i is chosen. Since the voting functions depend on the representation chosen, it is a natural question to ask whether the voting functions $\hat{\pi}^{1i}$ and $\hat{\pi}^{2i}$ associated with \hat{u}^i are continuous if π^{1i} and π^{2i} are continuous, or in other words is Assumption B2 independent of the representation chosen for the preference relation. To show this, let the function $h^i: \hat{u}^i(X^i) \rightarrow u^i(X^i)$ be defined by $h^i(\hat{u}^i(x^i)) = u^i(x^i)$, $\forall x^i \in X^i$. Let $(t^r)_{r \in \mathbb{N}}$ be a sequence in $\hat{u}^i(X^i)$ converging to a point $t \in \hat{u}^i(X^i)$. It is not difficult to show that it is possible to construct a sequence $(x^r)_{r \in \mathbb{N}}$ in X^i satisfying $\hat{u}^i(x^r) = t^r$ and $(x^r)_{r \in \mathbb{N}} \rightarrow x^i$ for some $x^i \in X^i$ with $\hat{u}^i(x^i) = t$. Then

$\lim_{r \rightarrow \infty} h^i(t^r) = \lim_{r \rightarrow \infty} h^i(\hat{u}^i(x^r)) = \lim_{r \rightarrow \infty} u^i(x^r) = u^i(x^i) = h^i(\hat{u}^i(x^i)) = h^i(t)$, and therefore the function h^i is continuous. Let $(v^{1i}, v^{2i}) \in \hat{v}^i(C^1) \times \hat{v}^i(C^2)$ be given, where \hat{v}^i is the indirect utility function associated with \hat{u}^i . Then $\hat{\pi}^{ki}(v^{1i}, v^{2i}) = \pi^{ki}(h^i(v^{1i}), h^i(v^{2i}))$ and hence $\hat{\pi}^{ki}$ is continuous by the continuity of the functions π^{ki} and h^i .

The political candidates are assumed to maximize either both their expected plurality or both their probability of winning the elections. In the first case the pay-off function $K^1: C^1 \times C^2 \rightarrow \mathbb{R}$ of political candidate 1 is defined by

$$K^1(c^1, c^2) = \sum_{i=1}^m \pi^{1i}(\bar{v}^i(c^1), \bar{v}^i(c^2)) - \sum_{i=1}^m \pi^{2i}(\bar{v}^i(c^1), \bar{v}^i(c^2)), \forall c^1 \in C^1, \forall c^2 \in C^2. \quad (4)$$

The pay-off function $K^2: C^1 \times C^2 \rightarrow \mathbb{R}$ of political candidate 2 is easily seen to be equal to $-K^1$. If political candidates maximize their probability of winning the elections then the pay-off function $K^1: C^1 \times C^2 \rightarrow \mathbb{R}$ of political candidate 1 is defined by

$$K^1(c^1, c^2) = \sum_{\{S \subset I_m \mid |S| \geq 1/2m + 1/2\}} \prod_{i \in S} \pi^{1i}(\bar{v}^i(c^1), \bar{v}^i(c^2)) \prod_{i \in I_m \setminus S} \pi^{2i}(\bar{v}^i(c^1), \bar{v}^i(c^2)) + \frac{1}{2} \sum_{\{S \subset I_m \mid |S| = 1/2m\}} \prod_{i \in S} \pi^{1i}(\bar{v}^i(c^1), \bar{v}^i(c^2)) \prod_{i \in I_m \setminus S} \pi^{2i}(\bar{v}^i(c^1), \bar{v}^i(c^2)) - \frac{1}{2}, \forall c^1 \in C^1, \forall c^2 \in C^2. \quad (5)$$

Here $|S|$ denotes the number of elements of a set S . Empty sets are included in the summation and the convention is made that $\prod_{i \in \emptyset} \pi^{ki}(\bar{v}^i(c^1), \bar{v}^i(c^2)) = 1$. Notice that the probability of political candidate 1 winning the elections is obtained by summation over all sets containing at least half of the consumers of the probability that all the consumers in this set vote for political candidate 1, while the consumers in the complement of this set vote for political candidate 2. In case of a tie, the toss with a fair coin determines the outcome of the elections. Notice that the subtraction of $\frac{1}{2}$ implies that the pay-off function $K^2: C^1 \times C^2 \rightarrow \mathbb{R}$ of political candidate 2 is given by $K^2 = -K^1$. Now the price rigidities ruling in the economic system will be determined endogenously as the price regulations resulting in a Nash equilibrium of a game with sets of admissible actions C^1 and C^2 and pay-off functions K^1 and K^2 , where K^1 and K^2 are either as defined by (4) or as defined by (5).

Definition 3.2 (Political Economic Equilibrium). *A political economic equilibrium of the political economic system $\mathcal{E} = (\{X^i, \geq^i, w^i\}_{i=1}^m, \{A^k, \{\pi^{ki}\}_{i=1}^m\}_{k=1}^2)$ is a Nash equilibrium for the mixed extension of the game $\mathcal{G} = \langle C^1, C^2, K^1, K^2 \rangle$, where C^1 and C^2 are as defined in (3), and K^1 and K^2 are either as defined by (4) or as defined by (5).*

For $k \in I_2$, let $D(C^k)$ be the set of Borel probability measures on C^k . Then, formally, a Nash equilibrium for the mixed extension of the game \mathcal{G} defined above is a pair of probability measures $(\mu^{*1}, \mu^{*2}) \in D(C^1) \times D(C^2)$

such that

$$\begin{aligned} & \int_{(c^1, c^2) \in C^1 \times C^2} K^1(c^1, c^2) d(\mu^{*1}(c^1) \times \mu^{*2}(c^2)) \\ &= \max_{\mu^1 \in D(C^1)} \int_{(c^1, c^2) \in C^1 \times C^2} K^1(c^1, c^2) d(\mu^1(c^1) \times \mu^{*2}(c^2)), \end{aligned}$$

and

$$\begin{aligned} & \int_{(c^1, c^2) \in C^1 \times C^2} K^2(c^1, c^2) d(\mu^{*1}(c^1) \times \mu^{*2}(c^2)) \\ &= \max_{\mu^2 \in D(C^2)} \int_{(c^1, c^2) \in C^1 \times C^2} K^2(c^1, c^2) d(\mu^{*1}(c^1) \times \mu^2(c^2)), \end{aligned}$$

see for example Dasgupta and Maskin [9].

Usually, theorists in voting theory are reluctant to use an equilibrium in mixed strategies as a solution to a game as described in Definition 3.2. The main objection against mixed strategies is that the game in Definition 3.2 does not take into account the dynamic features of campaigns in real world elections. During the campaign political candidates have the possibility to sequentially adjust their proposals. In case an equilibrium in pure strategies exists a political candidate can use his pure strategy during the entire campaign. In case political candidates adopt a mixed strategy then the proposals made by them at a specific point in time will in general not be best responses to each other. Therefore, on the next point in time a political candidate might want to adjust his strategy. However, as has been pointed out in McKelvey and Ordeshook [26], this criticism is not completely justified and it is interesting to consider mixed strategy equilibria too, their main reasons being the following. First, if one wants to analyze dynamic issues, then this should be explicitly incorporated in the game defined. Considering mixed strategy equilibria for the static game of Definition 3.2 is an essential first step. Second, it is possible to give several reasonable dynamic models where indeed equilibria corresponding to the mixed strategy equilibria of the static game are obtained. Moreover, it seems reasonable that in a dynamic context political candidates make proposals in the support set of mixed strategy equilibria of the static game.

4 Existence results

The political economic system $\hat{\mathcal{E}}$ is assumed to be given in this section. Sufficient conditions for both the existence of a political economic equilibrium in mixed strategies and the existence of a political economic equilibrium in pure strategies of the political economic system $\hat{\mathcal{E}}$ will be given. The first step is to show that the set of admissible actions of each political candidate is non-empty. The following theorem guarantees the existence of a Drèze equilibrium for given lower and upper bounds on the prices.

Theorem 4.1. *Let the economy $\mathcal{E}_{(p, \bar{p})}$ satisfy the Assumptions A1–A4. Then $E_{(p, \bar{p})}^D \neq \emptyset$.*

Proof. See Drèze [12].

In Theorem 4.2 it is shown that there exists $M \in \mathbb{R}_+$ such that, for every price regulation $(p, \bar{p}) \in R$, for every Drèze equilibrium $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E^D(p, \bar{p})$ it holds that if $p_j^* \geq M$ for some commodity $j \in I_n$, then every consumer keeps his initial endowments of this commodity. This means that no trade occurs on the market of this commodity. The number M can be chosen independently of the price regulations imposed. This is quite remarkable since the following intuition behind this result is wrong. If the price of a commodity is very high, then every consumer wants to supply this commodity and the only way to obtain a Drèze equilibrium is therefore by full rationing on supply on the market of this commodity. This intuition is not correct, since there might be prices of other commodities being as high, so it is impossible to guarantee that every commodity having a very high price is supplied by every consumer. The right argument involves equilibrium considerations and goes along the following lines. If the price of a commodity is very high and a consumer demands this commodity then supply rationing on this market is not binding for this consumer. Therefore, using Lemma 2.3, it can be shown that this consumer also demands an amount of the numeraire commodity exceeding the total initial endowments of the numeraire commodity, giving a contradiction since this cannot happen in a Drèze equilibrium. Consequently, in a Drèze equilibrium it does hold that no consumer demands a commodity having a sufficiently high price. Since in a Drèze equilibrium supply equals demand of every commodity, there is full rationing on supply of commodities having a sufficiently high price and every consumer keeps his initial endowments of these commodities. It has to be remarked that it is indeed possible that in a Drèze equilibrium the price exceeds M since the minimum price of a commodity could be greater than M . In case the assumptions of Theorem 4.2 are not satisfied it is possible that a number M with the desired properties does not exist. This follows immediately from the example in Section 6 of Bénassy [1]. The Drèze equilibria derived in this example satisfy the equilibrium concepts of both Bénassy and Drèze, while for every price regulation in R on every market some trade occurs in the corresponding Drèze equilibrium. In that example the preference relations of the consumers satisfy only weak monotonicity, and the initial endowments of every consumer are an element of the boundary of the consumption set, whereas the other assumptions made in Theorem 4.2 are satisfied.

Theorem 4.2. Let $\{X^i, \geq^i, w^i\}_{i=1}^m$ satisfy the Assumptions A1–A3. Let $M \in \mathbb{R}_+$ be as in Lemma 2.3. Then, for every $(p, \bar{p}) \in R$, for every Drèze equilibrium $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E^D_{(p, \bar{p})}$ for every $j \in I_n$ with $p_j^* \geq M$, $x_j^{*i} = w_j^i, \forall i \in I_m$, and $l_j^* = 0$.

Proof. Suppose $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E^D_{(p, \bar{p})}$ with for some $k \in I_n, p_k^* \geq M$ and $l_k^* < 0$. Moreover, let k be such that for every $j \in I_n, p_j^* > p_k^*$ implies $l_j^* = 0$. Suppose that l_k^* is binding for every consumer $i \in I_m$. Then it holds that $\sum_{i=1}^m (x_k^{*i} - w_k^i) = \sum_{i=1}^m l_k^* < 0$, a contradiction. Consequently, some consumer

$h \in I_m$ is not rationed on his supply on market k , and so $x^{*h} \in \Delta^h(\tilde{l}^*, L^*, p^*)$, with $\tilde{l}_k^* = -\bar{w}_k$ and $\tilde{l}_j^* = l_j^*, \forall j \in I_n \setminus \{k\}$. By Lemma 2.2 there is no loss of generality in assuming that for every $j \in I_n, l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j$. So $(\tilde{l}^*, L^*, p^*) \in \mathcal{L}_M$ and therefore, by Lemma 2.3, $x_0^{*h} > \bar{w}_0$, a contradiction. Consequently, for every $j \in I_n$ with $p_j^* \geq M$ it holds that $l_j^* = 0$ and, moreover, $x_j^{*i} \geq w_j^i, \forall i \in I_m$. Since $\sum_{i=1}^m x_j^{*i} = \sum_{i=1}^m w_j^i$ it follows that $x_j^{*i} = w_j^i, \forall i \in I_m$. Q.E.D.

Lemma 4.3 shows that there is a compact subset of R such that for every element of R outside this compact set, the set of corresponding Drèze equilibrium allocations is the same as the set of Drèze equilibrium allocations corresponding with some element of R in this compact set. This is not surprising since by Theorem 4.2 we know that there exists $M \in \mathbb{R}_+$ such that if the Drèze equilibrium price on some market is greater than or equal to M , then no trade takes place on this market and there is full rationing on supply on this market. By Condition 4 of Definition 2.1 it follows that the Drèze equilibrium price is equal to the lower bound on the price of this market. The basic idea of the proof is therefore to change this lower bound (and also the upper bound) to M and to show that the original Drèze equilibrium is also a Drèze equilibrium for the economy with the new lower and upper bounds, where Drèze equilibrium prices originally exceeding M are now equal to M . Since no trade occurred on the markets with prices exceeding M , the Drèze equilibrium consumption bundles are still in the budget sets of the consumers. The problem is to show that no consumer would be better off by having a positive excess demand of some of the commodities with prices originally exceeding M . By means of a quite technical proof, this can indeed be shown. It is very easy to show that no new Drèze equilibria are added by the procedure described above.

Lemma 4.3. *Let the economy $\mathcal{E}_{(p, \bar{p})}$ satisfy the Assumptions A1–A4 and let $M \in \mathbb{R}_+$ be as in Lemma 2.3. Let $(\tilde{p}, \bar{p}) \in R$ be given by $\tilde{p}_j = \min\{M, p_j\}, \forall j \in I_n$, and $\bar{p}_j = \min\{M, \bar{p}_j\}, \forall j \in I_n$. Then $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(p, \bar{p})}^D$ for some $p^* \in P_{(p, \bar{p})}$ if and only if $(x^{*1}, \dots, x^{*m}, l^*, L^*, \tilde{p}^*) \in E_{(\tilde{p}, \bar{p})}^D$ for some $\tilde{p}^* \in P_{(\tilde{p}, \bar{p})}$.*

Proof. Using Lemma 2.2 there is no loss of generality in assuming that for every $j \in I_n, l_j^* \geq -\bar{w}_j$ and $L_j^* \leq \bar{w}_j$.

Let $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*) \in E_{(p, \bar{p})}^D$ be given. Let $\tilde{p}^* \in P_{(\tilde{p}, \bar{p})}$ be defined by $\tilde{p}_j^* = \min\{M, p_j^*\}, \forall j \in I_n, \tilde{p}_0^* = 1$. Clearly, $(x^{*1}, \dots, x^{*m}, l^*, L^*, \tilde{p}^*)$ satisfies Conditions 2 and 3 of Definition 2.1. If $\tilde{p}_j^* < \tilde{p}_j$ then $p_j^* < \bar{p}_j$ and if $\tilde{p}_j^* > \tilde{p}_j$ then $p_j^* > \bar{p}_j$, and therefore Condition 4 of Definition 2.1 is satisfied. Let the set J be defined by $J = \{j \in I_n \mid \tilde{p}_j^* = M\}$. Then, $p_j^* \geq M, \forall j \in J$, and $\tilde{p}_j^* = p_j^*, \forall j \in I_n \setminus J$. Moreover, for every $j \in J, 0 = l_j^* = x_j^{*i} - w_j^i, \forall i \in I_m$, by Theorem 4.2, so $x^{*i} \in \Gamma^i(l^*, L^*, \tilde{p}^*)$. It remains to be shown that $x^{*i} \in \delta^i(l^*, L^*, \tilde{p}^*), \forall i \in I_m$. Suppose for some $h \in I_m$ there exists $y^h \in \delta^h(l^*, L^*, \tilde{p}^*)$ such that $y^h \succ^h x^{*h}$. Then, using the convexity of preferences, there exists $\hat{y}^h \in \Delta^h(l^*, L^*, \tilde{p}^*)$ such that $\hat{y}^h \succ^h x^{*h}$. Clearly, $\hat{y}^h \notin \Gamma^h(l^*, L^*, p^*)$, so $p^* \cdot (\hat{y}^h - w^h) > 0 \geq \tilde{p}^* \cdot (\hat{y}^h - w^h)$, and using that $p_j^* = \tilde{p}_j^*, \forall j \in I_n \setminus J, p_j^* \geq \tilde{p}_j^*, \forall j \in J$, and $l_j^* = 0, \forall j \in J$, it has to hold that $\hat{y}_k^h > w_k^h$ for some $k \in J$. For every $\lambda \in [0, 1]$, let $p(\lambda)$ be defined by

$p(\lambda) = \lambda \tilde{p}^* + (1 - \lambda)p^*$. Let $\lambda^* \in [0, 1]$ be defined by $\lambda^* = \sup\{\lambda \in [0, 1] \mid x^{*h} \in \Delta^h(l^*, L^*, p(\lambda))\}$. So there exists a sequence $(\lambda^r)_{r \in \mathbb{N}}$ such that $\lambda^r \rightarrow \lambda^*$, $0 \leq \lambda^r \leq \lambda^*$, and $x^{*h} \in \Delta^h(l^*, L^*, p(\lambda^r))$. Hence, $x^{*h} \in \Delta^h(l^*, L^*, p(\lambda^*))$ using the upper semi-continuity of Δ^h (since $p_0(\lambda^*) = 1$, see Drèze [12]) at the point $(l^*, L^*, p(\lambda^*))$. As a consequence of the existence of \hat{y}^h , $\lambda^* < 1$. Now let $(\lambda^r)_{r \in \mathbb{N}}$ be a sequence such that $\lambda^r \rightarrow \lambda^*$ and $\lambda^r > \lambda^*$. Then there exists $\hat{y}^{hr} \in \Delta^h(l^*, L^*, p(\lambda^r))$ such that $\hat{y}^{hr} >^h x^{*h}$, and similarly as before it follows that there exists $k^r \in J$ such that $\hat{y}^{hr} > w_{k^r}^h$. Hence, $\hat{y}^{hr} \in \Delta^h(\bar{l}^{*r}, L^*, p(\lambda^r))$ where $\bar{l}_{k^r}^{*r} = -\bar{w}_{k^r}$ and $\bar{l}_j^{*r} = l_j^*$, $\forall j \in I_n \setminus \{k^r\}$. Without loss of generality $(\hat{y}^{hr}, \bar{l}^{*r})_{r \in \mathbb{N}} \rightarrow (\bar{y}^h, \bar{l}^*)$, where for some $k \in J$, $\bar{l}_k^* = -\bar{w}_k$ and $\bar{l}_j^* = l_j^*$, $\forall j \in I_n \setminus \{k\}$. Moreover, $\bar{y}^h \in \Delta^h(\bar{l}^*, L^*, p(\lambda^*))$ using the upper semi-continuity of Δ^h at the point $(\bar{l}^*, L^*, p(\lambda^*))$. Since $\hat{y}^{hr} \in \Delta^h(l^*, L^*, p(\lambda^r))$ it follows also that $\bar{y}^h \in \Delta^h(l^*, L^*, p(\lambda^*))$. So $x^{*h} \sim^h \bar{y}^h$ and therefore $x^{*h} \in \Delta^h(\bar{l}^*, L^*, p(\lambda^*))$. Since $(\bar{l}^*, L^*, p(\lambda^*)) \in \mathcal{L}_{M, x_0^h} > \bar{w}_0$, a contradiction since $\sum_{i=1}^m x_0^{*i} = \bar{w}_0$. Consequently, $x^{*i} \in \delta^i(l^*, L^*, \tilde{p}^*)$, $\forall i \in I_m$.

Now let $(x^{*1}, \dots, x^{*m}, l^*, L^*, \tilde{p}^*) \in E_{(\tilde{p}, \tilde{p})}^D$ be given. Let the sets J^1 and J^2 be defined by

$$J^1 = \{j \in I_n \mid \tilde{p}_j^* = M \text{ and } \forall i \in I_m, l_j^* < x_j^{*i} - w_j^i\},$$

$$J^2 = \{j \in I_n \mid \tilde{p}_j^* = M \text{ and } \exists i \in I_m, l_j^* = x_j^{*i} - w_j^i\}.$$

Let $p^* \in P_{(p, \tilde{p})}$ be defined by $p_j^* = \tilde{p}_j$, $\forall j \in J^1$, $p_j^* = \underline{p}_j$, $\forall j \in J^2$, and $p_j^* = \tilde{p}_j^*$, $\forall j \in I_n^0 \setminus (J^1 \cup J^2)$. It is easily verified that $(x^{*1}, \dots, x^{*m}, l^*, L^*, p^*)$ satisfies Conditions 2, 3, and 4 of Definition 2.1. For $j \in I_n$, if $\tilde{p}_j^* < M$ then $p_j^* = \tilde{p}_j^*$, and if $\tilde{p}_j^* = M$ then $p_j^* \geq \tilde{p}_j^*$ and by Theorem 4.2, $0 = l_j^* = x_j^{*i} - w_j^i$, $\forall i \in I_m$. Consequently, for every $i \in I_m$, $x^{*i} \in \gamma^i(l^*, L^*, p^*) \subset \gamma^i(l^*, L^*, \tilde{p}^*)$, and hence $x^{*i} \in \delta^i(l^*, L^*, p^*)$. So Condition 1 of Definition 2.1 is satisfied. Q.E.D.

Let $M \in \mathbb{R}_+$ be as in Lemma 2.3. For $k \in I_2$, define a restricted set of admissible price regulations A_M^k of a political candidate $k \in I_2$ by $A_M^k = \beta_M^{2n}(A^k)$, i.e., A_M^k is obtained by means of an orthogonal projection of the elements of A^k on the cube β_M^{2n} . This projection yields exactly the procedure described above Lemma 4.3, i.e., lower bounds and upper bounds on prices exceeding M are replaced by M . Lemma 4.3 guarantees that the set of Drèze equilibrium allocations does not change by this procedure. Define a restricted set of admissible actions C_M^k of a political candidate $k \in I_2$ as in (3) where A^k is replaced by A_M^k . Then Lemma 4.3 guarantees that the strategic possibilities of political candidate k corresponding to the sets C_M^k and C^k are the same. Lemma 4.4 states that under the assumptions made, the set of actions C_M^k is non-empty and compact for $k \in I_2$. The compactness of A_M^k follows immediately from semi-compactness of A^k . The compactness of C_M^k then follows from the way the set of Drèze equilibria depends on the price regulations chosen. The non-emptiness of C_M^k follows immediately from the non-emptiness of A^k and the existence of a Drèze equilibrium given any price regulation.

Lemma 4.4. *Let $\{X^i, \geq^i, w^i\}_{i=1}^m, \{A^k\}_{k=1}^2$ satisfy the Assumptions A1–A3 and B1, and let $M \in \mathbb{R}_+$ be as in Lemma 2.3. Then the set $C_M^k, k \in I_2$, is non-empty and compact.*

Proof. For $k \in I_2$ it follows immediately from Theorem 4.1, Theorem 2.5, and the set A_M^k being non-empty according to Assumption B1, that the set C_M^k is non-empty. Clearly, the set C_M^k is bounded. The set C_M^k is closed if every convergent sequence in C_M^k has a limit in C_M^k . Let $(\underline{p}^r, \bar{p}^r, q^r)_{r \in \mathbb{N}}$ be a convergent sequence in C_M^k with limit $(\underline{p}, \bar{p}, q)$. By definition of C_M^k there exists a sequence $(x^r, \underline{p}^r, \bar{p}^r, q^r)_{r \in \mathbb{N}}$, such that $(x^r, \hat{l}(q^r), \hat{L}(q^r), \hat{p}(\underline{p}^r, \bar{p}^r, q^r)) \in E_{(\underline{p}^r, \bar{p}^r)}^D$. This sequence is obviously bounded and therefore has a convergent subsequence $(x^{r'}, \underline{p}^{r'}, \bar{p}^{r'}, q^{r'})_{r' \in \mathbb{N}}$ with limit, say, $(x, \underline{p}, \bar{p}, q)$. Since the sets X^1, \dots, X^m, Q , and A_M^k are closed (by Assumption B1), it holds that $x^i \in X^i, \forall i \in I_m, q \in Q$, and $(\underline{p}, \bar{p}) \in A_M^k$. It will be shown that $(x, \hat{l}(q), \hat{L}(q), \hat{p}(\underline{p}, \bar{p}, q)) \in E_{(\underline{p}, \bar{p})}^D$. Clearly, for $j \in I_n$,

$$\begin{aligned} \hat{p}_j(\underline{p}^{r'}, \bar{p}^{r'}, q^{r'}) &= \max \{ \underline{p}_j^{r'}, \min \{ \bar{p}_j^{r'}, \underline{p}_j^{r'}(2 - 3q_j^{r'}) + \bar{p}_j^{r'}(3q_j^{r'} - 1) \} \} \\ &\rightarrow \max \{ \underline{p}_j, \min \{ \bar{p}_j, \underline{p}_j(2 - 3q_j) + \bar{p}_j(3q_j - 1) \} \} = \hat{p}_j(\underline{p}, \bar{p}, q), \end{aligned} \tag{6}$$

$$\hat{l}_j(q^{r'}) = - \min \{ 1, 3q_j^{r'} \} \bar{w}_j \rightarrow - \min \{ \{ 1, 3q_j \} \bar{w}_j = \hat{l}_j(q), \tag{7}$$

$$\hat{L}_j(q^{r'}) = \min \{ 1, 3 - 3q_j^{r'} \} \bar{w}_j \rightarrow \min \{ 1, 3 - 3q_j \} \bar{w}_j = \hat{L}_j(q). \tag{8}$$

Using the results in Drèze [12] it follows that for every $i \in I_m, \delta^i$ is an upper semi-continuous correspondence from $-\mathbb{R}_+^n \times \mathbb{R}_+^n \times \{1\} \times \mathbb{R}_+^n$ into X^i . Using (6), (7), and (8), this yields $x^i \in \delta^i(\hat{l}(q), \hat{L}(q), \hat{p}(\underline{p}, \bar{p}, q))$. Consequently, Condition 1 is satisfied. Condition 2 holds since $\sum_{i=1}^m x^i = \sum_{i=1}^m \lim_{s \rightarrow \infty} x^{i^s} = \lim_{s \rightarrow \infty} \sum_{i=1}^m x^{i^s} = \bar{w}$. Obviously, $0 \leq x_j^i \leq \bar{w}_j, \forall i \in I_m, \forall j \in I_n$. This property will be used in order to prove that Conditions 3 and 4 are satisfied. Suppose there exists a consumer $i \in I_m$ and a commodity $j \in I_n$ such that $x_j^i - w_j^i = \hat{L}_j(q)$. Then $q_j > \frac{2}{3}$ and so $\hat{l}_j(q) = -\bar{w}_j$. Hence, for all consumers $h \in I_m$ it holds that $x_j^h - w_j^h > \hat{l}_j(q)$. Suppose now there exists a consumer $i \in I_m$ and a commodity $j \in I_n$ such that $x_j^i - w_j^i = \hat{l}_j(q)$. Then $q_j < \frac{1}{3}$ and so $\hat{L}_j(q) = \bar{w}_j$. Hence, for all consumers $h \in I_m$ it holds that $x_j^h - w_j^h < \hat{L}_j(q)$, which shows that Condition 3 is satisfied. Suppose there exists a commodity $j \in I_n$ such that $\hat{p}_j(\underline{p}, \bar{p}, q) < \bar{p}_j$. Then $q_j < \frac{2}{3}$ and consequently $\hat{L}_j(q) = \bar{w}_j > x_j^i - w_j^i, \forall i \in I_m$. Finally, suppose there exists a commodity $j \in I_n$ such that $\hat{p}_j(\underline{p}, \bar{p}, q) > \underline{p}_j$. Then $q_j > \frac{1}{3}$ and consequently $\hat{l}_j(q) = -\bar{w}_j < x_j^i - w_j^i, \forall i \in I_m$, which proves that Condition 4 is satisfied. So, $(x, \hat{l}(q), \hat{L}(q), \hat{p}(\underline{p}, \bar{p}, q)) \in E_{(\underline{p}, \bar{p})}^D$. Since $(\underline{p}, \bar{p}) \in A_M^k$ it holds that $(\underline{p}, \bar{p}, q)$ is an element of C_M^k . Q.E.D.

Let $M \in \mathbb{R}_+$ be as in Lemma 2.3. For $k \in I_2$ define the restricted pay-off function of political candidate $k, K_M^k: C_M^1 \times C_M^2 \rightarrow \mathbb{R}$ as in (4) or (5). Since Lemma 4.3 guarantees that the strategic possibilities of political candidate k corresponding to the sets C_M^k and C^k are the same, i.e., the set of Drèze equilibrium allocations corresponding to the sets C_M^k and C^k are the same, it holds that $\bar{v}^i(C^k) = \bar{v}^i(C_M^k)$, so the voting function $\pi^{ki}: \bar{v}^i(C^1) \times \bar{v}^i(C^2) \rightarrow [0, 1]$ is well defined on $\bar{v}^i(C_M^1) \times \bar{v}^i(C_M^2)$. Therefore, the restricted pay-off function of political candidate k, K_M^k , is well defined on $C_M^1 \times C_M^2$. In Lemma 4.5 it is shown that the function K_M^k is continuous. This property is directly related to the continuity of the utility functions and the voting functions.

Lemma 4.5. *Let the political economic system $\hat{\mathcal{E}}$ satisfy the Assumptions A1–A3 and B1–B2. Let $M \in \mathbb{R}_+$ be as in Lemma 2.3. Then for $k \in I_2$ the function $K_M^k: C_M^1 \times C_M^2 \rightarrow \mathbb{R}$ as defined in (4) or (5) is continuous.*

Proof. It is first shown that the function $\bar{v}^i, \forall i \in I_m$, is continuous on C . Let $(p^r, \bar{p}^r, q^r)_{r \in \mathbb{N}}$ be a sequence in C converging to an element (p, \bar{p}, q) . For every $r \in \mathbb{N}$, let x^r be an element of $\delta^i(\hat{l}(q^r), \hat{L}(q^r), \hat{p}(p^r, \bar{p}^r, q^r))$ then, similar to the proof of Lemma 4.4, without loss of generality, the sequence $(x^r)_{r \in \mathbb{N}}$ converges to an element x^i and $x^i \in \delta^i(\hat{l}(q), \hat{L}(q), \hat{p}(p, \bar{p}, q))$. Hence, using the continuity of $u^i, \bar{v}^i(p^r, \bar{p}^r, q^r) = u^i(x^r) \rightarrow u^i(x^i) = \bar{v}^i(p, \bar{p}, q)$. By Lemma 4.3, $\bar{v}^i(C^k) = \bar{v}^i(C_M^k), \forall i \in I_m, \forall k \in I_2$, and therefore the function π^{ki} is continuous on $\bar{v}^i(C_M^1) \times \bar{v}^i(C_M^2)$ by Assumption B2. Now the continuity of K_M^k follows from the continuity of \bar{v}^i and $\pi^{ki}, \forall i \in I_m$. Q.E.D.

Theorem 4.6 states that there exists a political economic equilibrium, i.e., there exists a Nash equilibrium for the game where political candidates choose an admissible price regulation and a corresponding Drèze equilibrium pseudo-price, and where the pay-offs of the political candidates are determined by either the expected majority or the probability of winning the elections. Using the compactness of the sets of admissible actions, Lemma 4.4, and the continuity of the pay-off functions, Lemma 4.5, this result now follows from standard theory.

Theorem 4.6. *Let the political economic system $\hat{\mathcal{E}}$ satisfy the Assumptions A1–A3 and B1–B2. Then there exists a political economic equilibrium of the political economic system $\hat{\mathcal{E}}$.*

Proof. The existence of a Nash equilibrium of the mixed extension of the game $\mathcal{G} = \langle C^1, C^2, K^1, K^2 \rangle$ has to be shown. Let M be as in Lemma 2.3 and consider the game $\mathcal{G}_M = \langle C_M^1, C_M^2, K_M^1, K_M^2 \rangle$. According to Lemma 4.4 the sets C_M^1 and C_M^2 are non-empty and compact. By Lemma 4.5 K_M^1 and K_M^2 are continuous functions on $C_M^1 \times C_M^2$. Hence, using a theorem of Glicksberg [15], the mixed extension of \mathcal{G}_M has at least one Nash equilibrium outcome. Obviously, using Lemma 4.3, this Nash equilibrium outcome yields a Nash equilibrium outcome for the game \mathcal{G} . Q.E.D.

It is interesting to have sufficient conditions for the existence of a Nash equilibrium in pure strategies. Usually it is sufficient to assume certain concavity and convexity conditions with respect to the pay-off functions and strategy sets in order to prove the existence of a pure strategy Nash equilibrium, see for example Feldman and Lee [13]. However, since the set C^k is not necessarily convex these conditions might not be satisfied. In Theorem 4.7 other sufficient conditions for the existence of a pure strategy political economic equilibrium are given. It is clear that these conditions are very strong, since the following separability assumption is needed.

B3. For every consumer $i \in I_m$, there exists a function $f^i: \bar{v}^i(C^1) \rightarrow \mathbb{R}$ and a function $g^i: \bar{v}^i(C^2) \rightarrow \mathbb{R}$ such that $\pi^{1i}(v^{1i}, v^{2i}) = f^i(v^{1i}) - g^i(v^{2i}) + \frac{1}{2}, \forall (v^{1i}, v^{2i}) \in \bar{v}^i(C^1) \times \bar{v}^i(C^2)$.

Clearly, for all $i \in I_m, \pi^{2i}(v^{1i}, v^{2i}) = 1 - \pi^{1i}(v^{1i}, v^{2i}) = \frac{1}{2} + g^i(v^{2i}) - f^i(v^{1i}), \forall (v^{1i}, v^{2i}) \in \bar{v}^i(C^1) \times \bar{v}^i(C^2)$ and hence π^{2i} also satisfies the separability assumption. It is not difficult to show that the continuity of π^{1i} implies that f^i and g^i must be continuous.

Since the voting functions depend on the utility representation chosen for the preference relation of a consumer, it is important to show that Assumption B3 holds independent of this representation. Let \hat{u}^i be a representation of \succeq^i of consumer $i \in I_m$ different from u^i and define the function $h^i: \hat{u}^i(X^i) \rightarrow u^i(X^i)$ in the same way as has been done below Assumption B2. Denote the indirect utility functions and the voting functions corresponding with \hat{u}^i by $\hat{v}^i, \hat{\pi}^{1i},$ and $\hat{\pi}^{2i},$ respectively, for every $i \in I_m.$ Let $(v^{1i}, v^{2i}) \in \hat{\delta}^i(C^1) \times \hat{\delta}^i(C^2)$ be given. Then $\hat{\pi}^{1i}(v^{1i}, v^{2i}) = \pi^{1i}(h^i(v^{1i}), h^i(v^{2i})) = f^i(h^i(v^{1i})) - g^i(h^i(v^{2i})) + \frac{1}{2},$ so $\hat{\pi}^{1i}$ satisfies Assumption B3.

Theorem 4.7 states that there exists a Nash equilibrium in pure strategies for the game where political candidates choose an admissible price regulation and a corresponding Dr ze equilibrium pseudo-price, and where the pay-offs of the political candidates are determined by the expected majority if the Assumptions A1–A3 and B1–B3 are satisfied. Although this result will not be shown for the case where the pay-offs of political candidates are determined by the probability of winning the elections, it is also of importance for this case since Hinich [19] has argued that both objectives are the same if the number of voters tends to infinity.

Theorem 4.7. *Let the political economic system $\hat{\mathcal{E}}$ satisfy the Assumptions A1–A3 and B1–B3. Then there exists a pure strategy political economic equilibrium of the political economic system $\hat{\mathcal{E}}$ in case for $k \in I_2$ the function K^k is defined as in (4).*

Proof. Let $c^{*1} \in C^1$ be such that $\sum_{i=1}^m f^i(\bar{v}^i(c^{*1})) = \max_{c^1 \in C^1} \sum_{i=1}^m f^i(\bar{v}^i(c^1)),$ and let $c^{*2} \in C^2$ be such that $\sum_{i=1}^m g^i(\bar{v}^i(c^{*2})) = \max_{c^2 \in C^2} \sum_{i=1}^m g^i(\bar{v}^i(c^2)).$ Then c^{*1} and c^{*2} are well defined using Lemma 4.3, Assumption B1, and the continuity for every $i \in I_m$ of the functions $f^i, g^i,$ and $\bar{v}^i.$ Moreover,

$$\begin{aligned} K^1(c^{*1}, c^{*2}) &= 2 \sum_{i=1}^m f^i(\bar{v}^i(c^{*1})) - 2 \sum_{i=1}^m g^i(\bar{v}^i(c^{*2})) \\ &\geq 2 \sum_{i=1}^m f^i(\bar{v}^i(c^1)) - 2 \sum_{i=1}^m g^i(\bar{v}^i(c^{*2})), \forall c^1 \in C^1, \\ K^2(c^{*1}, c^{*2}) &= 2 \sum_{i=1}^m g^i(\bar{v}^i(c^{*2})) - 2 \sum_{i=1}^m f^i(\bar{v}^i(c^{*1})) \\ &\geq 2 \sum_{i=1}^m g^i(\bar{v}^i(c^2)) - 2 \sum_{i=1}^m f^i(\bar{v}^i(c^{*1})), \forall c^2 \in C^2. \end{aligned}$$

Hence, (c^{*1}, c^{*2}) is a pure strategy political economic equilibrium of the political economic system $\hat{\mathcal{E}}.$ Q.E.D.

Although Assumption B3 is very restrictive, it is of some interest since the voting model given in Coughlin, Mueller and Murrell [5] satisfies this assumption. Let $(c^1, c^2) \in C^1 \times C^2$ be given. In the model of Coughlin, Mueller and Murrell [5], a consumer $i \in I_m$ votes for political candidate 1 if $\bar{v}^i(c^1) - \bar{v}^i(c^2) > b^i$, does not vote if $\bar{v}^i(c^1) - \bar{v}^i(c^2) = b^i$, and votes for political candidate 2 if $\bar{v}^i(c^1) - \bar{v}^i(c^2) < b^i$, where the information of the political candidates is that b^i is a random variable being uniformly distributed in some given interval $[-r^i, r^i] \subset \mathbb{R}$. In this way it can be modelled that there is a bias in favour of or against a political candidate in the preferences of the voters, whereas this bias is not perfectly known to the political candidates. It is assumed that $|\bar{v}^i(c^1) - \bar{v}^i(c^2)| \leq r^i, \forall (c^1, c^2) \in C^1 \times C^2$. This implies, for every $i \in I_m$,

$$\pi^{1i}(c^1, c^2) = \frac{1}{2} + \frac{1}{2r^i}(\bar{v}^i(c^1) - \bar{v}^i(c^2)), \forall (c^1, c^2) \in C^1 \times C^2,$$

$$\pi^{2i}(c^1, c^2) = \frac{1}{2} + \frac{1}{2r^i}(\bar{v}^i(c^2) - \bar{v}^i(c^1)), \forall (c^1, c^2) \in C^1 \times C^2,$$

so in this case the choice $f^i(v^{1i}) = \frac{1}{2r^i}v^{1i}, \forall v^{1i} \in \bar{v}^i(C^1)$, and $g^i(v^{2i}) = \frac{1}{2r^i}v^{2i}, \forall v^{2i} \in \bar{v}^i(C^2)$, satisfies Assumption B3.

5 An example

In this section an example of the model of the previous sections will be examined. The example presents an extremely simple political economic system since it is an example with only two commodities and two consumers. Nevertheless, it is useful to illustrate the logic of the model. Moreover, even in this extremely simple example it is possible to obtain politically more desired allocations than the Walrasian equilibrium allocation by means of the imposition of price regulations.

First the set of Drèze equilibrium allocations in an economy will be considered given any price regulation. Consider an economy with two consumers, two commodities, and Cobb-Douglas utility functions. Commodity 0 is a numeraire commodity with price equal to one. The consumers have the following characteristics, $X^1 = X^2 = \mathbb{R}_+^2, u^1(x_0, x_1) = (x_0)^{1/5}(x_1)^{4/5}, \forall x \in X^1, u^2(x_0, x_1) = (x_0)^{4/5}(x_1)^{1/5}, \forall x \in X^2, w^1 = w^2 = (4, 1)$. Let some $(\underline{p}, \bar{p}) \in R$ be given such that $0 < \underline{p} = \bar{p} = p$. It is again useful to work with pseudo-prices. The rationing scheme will be a function of the pseudo-price q , where q is an element of the set $Q = [0, 1]$. The functions \hat{l} and \hat{L} are given by

$$\hat{l}(q) = -\min\{1, 2q\}\bar{w}_1 = -2\min\{1, 2q\}, \forall q \in Q,$$

$$\hat{L}(q) = \min\{1, 2 - 2q\}\frac{\bar{w}_0}{p} = \frac{8}{p}\min\{1, 2 - 2q\}, \forall q \in Q.$$

Notice that the functions \hat{l} and \hat{L} are slightly different from the ones used before. In case of a fixed price system the functions given above are easier to work with. It should also be mentioned that the utility functions u^1 and u^2 are not strongly monotonic. However, strong monotonicity does hold when the utility function u^i of a consumer $i \in I_2$ is restricted to the non-empty, closed, convex set $\bar{X}^i = \{x^i \in X^i \mid u^i(x^i) \geq u^i(\frac{1}{2}w^i)\}$ satisfying that $\bar{X}^i \subset \mathbb{R}_+^2$, $\bar{X}^i + \mathbb{R}_+^2 \subset \bar{X}^i$, and $w^i \in \text{int}(\bar{X}^i)$. Considering \bar{X}^i instead of X^i , it follows that the Assumptions A1–A3 are satisfied in this example. After some computations it follows that

$$\delta^1(\hat{l}(q), \hat{L}(q), (1, p)) = \begin{cases} \left(\frac{p+4}{5}, \frac{4p+16}{5p}\right), & 0 < p, \max\left\{0, \frac{p-16}{20p}\right\} \leq q \leq \min\left\{1, \frac{p+64}{80}\right\}, \\ \left(16q - 12, \frac{16(1-q)}{p} + 1\right), & 0 < p \leq 16, \frac{p+64}{80} \leq q \leq 1, \\ (4pq + 4, 1 - 4q), & 16 \leq p, 0 \leq q \leq \frac{p-16}{20p}, \end{cases}$$

and

$$\delta^2(\hat{l}(q), \hat{L}(q), (1, p)) = \begin{cases} \left(\frac{4p+16}{5}, \frac{p+4}{5p}\right), & 0 < p, \max\left\{0, \frac{p-1}{5p}\right\} \leq q \leq \min\left\{1, \frac{p+19}{20}\right\}, \\ \left(16q - 12, \frac{16(1-q)}{p} + 1\right), & 0 < p \leq 1, \frac{p+19}{20} \leq q \leq 1, \\ (4pq + 4, 1 - 4q), & 1 \leq p, 0 \leq q \leq \frac{p-1}{5p}. \end{cases}$$

By solving $\sum_{i=1}^2 \delta^i(\hat{l}(q), \hat{L}(q), (1, p)) = \bar{w}$ all pseudo-prices corresponding with a Drèze equilibrium for the economy $\mathcal{E}_{(p,p)}$ are obtained. Let the set of admissible price regulations for both political candidates be some non-empty subset \bar{A} of R such that $0 < a_1 = a_2, \forall a \in \bar{A}$. It follows that the set \bar{C} of admissible actions corresponding with the set of admissible price regulations \bar{A} is given by

$$\begin{aligned} \bar{C} = \{ & (p, p, q) \in \bar{A} \times Q \mid 0 < p \leq 1 \text{ and } q = 1, \text{ or } 1 < p < 4 \text{ and } q = \frac{21-p}{20}, \\ & \text{or } p = 4 \text{ and } \frac{3}{20} \leq q \leq \frac{17}{20}, \text{ or } 4 < p < 16 \text{ and } q = \frac{16-p}{20p}, \\ & \text{or } p \geq 16 \text{ and } q = 0\}. \end{aligned} \tag{9}$$

Notice that $E_{(p,p)}^D \neq \emptyset, \forall (p, p) \in \bar{A}$, as stated by Theorem 4.1. Although there is an interval of Drèze equilibrium pseudo-prices in the case where $p = 4$, they all correspond to the same Drèze equilibrium allocation, which is the unique Walrasian equilibrium allocation. Therefore, considering the remarks made at Theorem 2.4, the possibility of not specifying price regulations is not excluded. Moreover, there is no loss of generality in considering only the admissible action corresponding with $q = \frac{1}{2}$ in this case. It can be verified that in case

$0 < p \leq 1$ or $p \geq 16$ every consumer keeps his initial endowments in a Drèze equilibrium corresponding with these values of p . If $0 < p \leq 1$ then the price of commodity 1 is so low that both consumers want to supply the numeraire commodity in exchange for it. The only way to obtain a Drèze equilibrium is by means of full rationing on demand on the market of commodity 1. If $p \geq 16$ then both consumers want to supply commodity 1 in exchange for the numeraire commodity. Now the only way to obtain a Drèze equilibrium is by means of full rationing on supply on the market of commodity 1. So, $M = 16$ satisfies the requirements of Lemma 4.2. Notice that also Lemma 4.3 is satisfied for $M = 16$. Clearly, political candidates do not have any objective to propose price regulations being this extreme, and there is no loss of strategic possibilities for the political candidates if they restrict themselves to price regulations satisfying $1 \leq p \leq 16$.

Using the functions u^i and δ^i it is easy to derive the indirect utility function of consumer $i \in I_2$, $\bar{v}^i: \bar{C} \rightarrow \mathbb{R}$. The voting functions of both consumers are assumed to be the same as in Coughlin and Nitzan [6], i.e., for every $i \in I_2$, for every $k \in I_2$,

$$\pi^{ki}(v^{1i}, v^{2i}) = \frac{e^{v^{ki}}}{e^{v^{1i}} + e^{v^{2i}}}, \forall v^{1i}, v^{2i} \in \bar{v}^i(\bar{C}).$$

Notice that the Assumption B2 is satisfied. Therefore, if the set of admissible price regulations \bar{A} of the political candidates satisfies Assumption B1, then a political economic equilibrium of the political economic system \mathcal{E} exists according to Theorem 4.6.

Suppose political candidates attempt to maximize their expected plurality. It is easy to show that for every $(c^1, c^2) \in \bar{C} \times \bar{C}$ it holds that

$$\sum_{i=1}^2 \pi^i(\bar{v}^i(c^1), \bar{v}^i(c^2)) - \sum_{i=1}^2 \pi^i(\bar{v}^i(c^1), \bar{v}^i(c^2)) \geq 0 \Leftrightarrow e^{\bar{v}^1(c^1)} e^{\bar{v}^2(c^1)} \geq e^{\bar{v}^1(c^2)} e^{\bar{v}^2(c^2)}.$$

Now suppose political candidates attempt to maximize their probability of winning the elections. Then it is easy to show that

$$\pi^{11}(\bar{v}^1(c^1), \bar{v}^1(c^2))\pi^{12}(\bar{v}^2(c^1), \bar{v}^2(c^2)) + \frac{1}{2}\pi^{11}(\bar{v}^1(c^1), \bar{v}^1(c^2))\pi^{22}(\bar{v}^2(c^1), \bar{v}^2(c^2)) + \frac{1}{2}\pi^{12}(\bar{v}^2(c^1), \bar{v}^2(c^2))\pi^{21}(\bar{v}^1(c^1), \bar{v}^1(c^2)) - \frac{1}{2} \geq 0 \Leftrightarrow e^{\bar{v}^1(c^1)} e^{\bar{v}^2(c^1)} \geq e^{\bar{v}^1(c^2)} e^{\bar{v}^2(c^2)}.$$

Using the symmetry of the game, it is then easily seen that both in the case where political candidates maximize expected plurality and in the case where political candidates maximize their probability of winning the elections, in a political economic equilibrium both political candidates choose an action $c^* \in \bar{C}$ that maximizes $\sum_{i=1}^2 \bar{v}^i(c)$ over $c \in \bar{C}$, i.e., both political candidates choose an admissible action that maximizes the sum of the indirect utilities of the consumers.

Consider the case where $\bar{A} = \{(3, 3), (4, 4), (5, 5)\}$. Hence, the set \bar{C} is given by $\{(3, 3, \frac{9}{10}), (4, 4, \frac{1}{2}), (5, 5, \frac{100}{10})\}$. Clearly, Assumption B1 is satisfied. The Drèze equilibria of the economy $\mathcal{E}_{(p,p)}$ for $p = 3$, $p = 4$, and $p = 5$ are summarized in Table I. Choosing $p = 3$, i.e., a value below the Walrasian equilib-

Table 1 Drèze equilibria.

	$p = 3$	$p = 4$	$p = 5$
q	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{11}{100}$
$\delta^1(\hat{l}(q), \hat{L}(q), (1, p))$	$(2\frac{2}{5}, 1\frac{8}{15})$	$(1\frac{3}{5}, 1\frac{3}{5})$	$(1\frac{4}{5}, 1\frac{11}{25})$
$\delta^2(\hat{l}(q), \hat{L}(q), (1, p))$	$(5\frac{3}{5}, \frac{7}{5})$	$(6\frac{2}{5}, \frac{2}{5})$	$(6\frac{1}{5}, \frac{14}{25})$
$\hat{l}(q)$	-2	-2	$-\frac{11}{25}$
$\hat{L}(q)$	$\frac{8}{15}$	2	$\frac{8}{5}$
$\bar{v}^1(p, p, q)$	1.677	1.6	1.506
$\bar{v}^2(p, p, q)$	3.407	3.676	3.833
$\sum_{i=1}^2 \bar{v}^i(p, p, q)$	5.084	5.276	5.339

rium price, is beneficial for consumer 1, since he demands commodity 1, and therefore has to spend less resources now to obtain this demand. It is detrimental for consumer 2, since his marginal utility of consuming commodity 1 is low and he is therefore a supplier of commodity 1. At $p = 3$ there is a positive total excess demand on the market of commodity 1, and therefore a Drèze equilibrium is sustained by means of demand rationing on this market. This has no consequences for consumer 2, since he supplies commodity 1. However, the demand rationing is binding for consumer 1. This rationing partially, but not completely, offsets his gain in utility. It is easily verified that, compared to the Walrasian equilibrium, the loss in utility of consumer 2 exceeds the gain in utility of consumer 1.

Choosing $p = 5$, i.e., a value above the Walrasian equilibrium price, is detrimental for consumer 1, since he demands commodity 1, and is beneficial for consumer 2, the supplier of commodity 1. At $p = 5$ there is a negative total excess demand on the market of commodity 1, and therefore a Drèze equilibrium is sustained by means of supply rationing on this market. This has no consequences for consumer 1, since he demands commodity 1. However, the supply rationing is binding for consumer 2. This rationing partially, but not completely, offsets his gain in utility. However, now, compared to the Walrasian equilibrium, the gain in utility of consumer 2 exceeds the loss in utility of consumer 1. Therefore, in the unique political economic equilibrium both political candidates choose for a price regulation where $p = 5$. It should be remarked that the action where $\underline{p} = 5$ and no maximum price is specified is equivalent in this case. So, in the political economic equilibrium, both political candidates specify a minimum price on the market of commodity 1, which excludes the Walrasian equilibrium price.

Now consider the case where $A^1 = A^2 = R$. It has already been remarked that the set of admissible actions \bar{C} corresponding with the set of admissible price regulations $\{(p, \bar{p}) \in R \mid 1 \leq \underline{p} = \bar{p} \leq 16\}$ gives each political candidate the same strategic possibilities as the set of admissible actions corresponding with

the set R . In a political economic equilibrium both political candidates therefore choose an action c^* such that $c^* = \arg \max_{c \in C} \sum_{i=1}^2 \bar{v}^i(c)$. Clearly, $c^* \neq (4, 4, \frac{1}{2})$. So, in the case where every price regulation is allowed, a price regulation is chosen where the Walrasian equilibrium price system and the Walrasian equilibrium allocation is excluded. It can be shown that $c^* = (5.035, 5.035, 0.1089)$. The optimal price regulation, being proposed by both political candidates in the unique political economic equilibrium, corresponds to the imposition of a minimum price on the market of commodity 1, excluding the Walrasian equilibrium price system. This yields a gain in utility for consumer 2, the supplier of commodity 1, who obtains a more favourable rate of exchange, but as a consequence of this will be rationed on his supply. Nevertheless, his gain in utility more than offsets the loss in utility of consumer 1.

Grandmont [16] explains the occurrence of temporary price rigidities and quantity rationing by making the observation that in the short run quantities move faster than prices. The example considered in this section demonstrates that price rigidities may exist in the long run too.

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