

On the Connections Between Walrasian and Rational Expectations Equilibria*

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Abstract

Using an implementation approach, we investigate the connections between Walrasian and Rational Expectations equilibria. We construct an elementary mechanism (Dutta, Sen and Vohra (1995)) that Nash implements the Constrained Walrasian correspondence. We extend it to incomplete and non-exclusive information economies by enlarging the message space of agents. We characterize the set of Bayesian equilibrium outcomes of the mechanism, and thus characterize an extension of the Constrained Walrasian correspondence to economies with differential information. Measurability restrictions on allocations with respect to prices do not emerge from the strategic behavior of agents: there exist simple economies for which the set of Constrained Rational Expectations equilibrium allocations is not contained in the set of equilibrium outcomes of the mechanism.

Keywords: Implementation, Elementary mechanisms, Walrasian equilibrium, Rational Expectations equilibrium.

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1 Introduction

The notion of a *rational expectations equilibrium* has been suggested as extension of *Walrasian equilibrium* to economies with differential information –see Radner (1979).¹ There is a vast literature on the game-theoretic foundations of, on the one hand WE, and on the other hand REE.² Starting with the seminal paper of Dubey, Geanakoplos and Shubik (1987), a recent literature on market games addresses the game-theoretic connections between Walrasian and rational expectations equilibrium. For instance, Forges and Minelli (1997), or more recently Codognato and Ghosal (2003) extend the well-known Shapley-Shubik strategic market games (see Shapley and Shubik (1977)) to differential information economies.³ In particular, Forges and Minelli (1997) obtain that when there is a continuum of agents of finitely many types, REE can be obtained as particular communication equilibria of the Shapley-Shubik market game.⁴

Using an implementation approach, we want to understand the game theoretical connections between (constrained) WE and (constrained) REE (henceforth CWE and CREE, respectively).⁵ However, we use a trading procedure that has a higher degree of centralization than Shapley-Shubik market games, as is standard in the implementation literature. A different line of research on decentralized trading procedures studies also the game-

¹From now on, we refer to rational expectations equilibrium as REE, and Walrasian equilibrium as WE.

²For the implementation of the Walrasian correspondence, see Hurwicz (1979), Schmeidler (1980), Postlewaite and Wettstein (1989), Dutta, Sen and Vohra (1995), or Bochet (2006) among others. Concerning the implementation of the rational expectations correspondence, see Palfrey and Srivastava (1987), Blume and Easley (1990), Wettstein (1990) and Bochet (2006) among others.

³Dubey, Geanakoplos and Shubik (1987) use a bids-offers market game while Forges and Minelli (1997) use a bids only market game. Codognato and Ghosal (2003) use a variant of the bids only market game in which it is not assumed that one specific commodity is used as money.

⁴Formally, the class of communication equilibria they study is called Self-fulfilling equilibria.

⁵The difference between Walrasian equilibria (respectively Rational expectations equilibria) and Constrained Walrasian equilibria (respectively Constrained Rational expectations equilibria) is that in the latter, agents maximize their utility with respect to constrained budget sets which exclude bundles that exceed the aggregate endowment. As a consequence, every Walrasian equilibrium (respectively Rational expectations equilibrium) is a Constrained Walrasian equilibrium (respectively Constrained Rational expectations equilibria) but the converse is obviously not true.

theoretic connections between WE and REE, and the strategic foundations of REE. For instance, see Gale (1987), Wolinsky (1990), or more recently Serrano (2002), Gottardi and Serrano (2005).⁶

We start by designing a mechanism that implements the CWE correspondence in Nash equilibrium. Our construction falls in the attractive class of elementary mechanisms, as defined in Dutta, Sen and Vohra (1995).^{7,8} Elementary mechanisms are an answer to what sort of message spaces are needed in order to implement the CWE correspondence. Our mechanism is based on the Walrasian notion of allocation, prices and “moves” along price hyperplanes which are central in the story behind the CWE correspondence. Similarly to the research question started by Dubey, Geanakoplos and Shubik (1987), we study how this mechanism –that performed well for a particular information structure and for a large class of economies– would work when extended to differential information economies.

We extend our mechanism to incomplete information by enlarging the message space of agents. Blume and Easley (1990) show that if non-exclusive information –henceforth NEI– is not satisfied, one can construct a robust example of an economy with a unique REE that is not incentive-compatible. Hence, we restrict our attention to non-exclusive differential information environments. We characterize the set of Bayesian equilibrium outcomes of the mechanism and thus suggest an extension of the CWE correspondence for differential information economies. We find that equilibrium allocation rules are expected utility maximizing over (constrained) state-contingent budget sets. We show that despite the similitude of the equilibrium outcomes set with CWE and CREE, the set of Bayesian Equilibrium outcomes of the mechanism does not always contain the set of CREE allocations.⁹ The measurability restrictions on allocations with respect to prices do not emerge from the strategic behavior of agents.¹⁰ An explanation for this is the follow-

⁶Although Gale’s model deals with a situation of complete information, his results extends to differential economies with private values.

⁷It is well-known that the Walrasian correspondence violates in general Maskin monotonicity (Maskin, 1999), a necessary condition for Nash implementation when corner allocations are possible. Instead of restricting the domain of economies and ruling out corner Walrasian allocations, we consider the Constrained Walrasian correspondence.

⁸Thereafter, we will make use of DSV to refer to Dutta, Sen and Vohra (1995).

⁹Namely, there exists economies for which the set of CREE allocations is not achievable as Bayesian equilibrium outcome of the mechanism.

¹⁰Suppose there are two states t and t' at which an agent i is of the same type t_i . If the prices in both states are the same, *measurability of allocations with respect to prices*

ing. At a REE, the only information that an agent obtains, on top of his own private information, is the one contained in the equilibrium prices. In the mechanism we use, on the other hand, there is a higher degree of centralization: an uninformed agent may propose an allocation based on information that he does not have, even after observing the aggregate trade or the equilibrium prices. This makes sense here because information is non-exclusive. The mechanism can easily extract the information held by the other individuals. Therefore, unless strong restrictions are imposed on the behavior of agents, it is not possible to obtain the equivalence between WE and REE – see also Bochet (2007). This result may cast some doubts on the connections between WE and REE beyond anonymous and fully decentralized market settings.

The plan of the paper is the following. In section 2, we present the model and the benchmark result for complete information settings. In section 3, we extend the presentation of the model to economies with differential information and present the main results. Finally, we provide some final comments in section 4.

2 Complete Information

There is a set $N = \{1, \dots, n\}$, $n \geq 3$, of *agents* and a set $L = \{1, \dots, \ell\}$ of *infinitely divisible goods*. The *consumption set* of each $i \in N$ is $X_i = \mathbb{R}_+^\ell$. *Preferences* of each $i \in N$ are represented by a *utility function* $u_i : X_i \rightarrow \mathbb{R}$. The *endowment* of each agent $i \in N$ is¹¹ $\omega_i > 0$. The *aggregate endowment* is denoted by $\bar{\omega} \gg 0$.

The only characteristics unknown to the planner are the utility functions. Individual endowments and the consumption sets of agents are known and fixed. An economy E is simply a profile of utility functions, one for each agent. More formally, $E = (u_i)_{i \in N}$. Denote by \mathcal{E} the *class of economies* in which, for each $i \in N$, u_i is continuous, strictly increasing and quasi-concave.

A (*feasible*) *allocation* is a list of *bundles* $x = (x_i)_{i \in N} \in \mathbb{R}_+^{\ln}$ such that $\sum x_i \leq \bar{\omega}$. Formally, the set of feasible allocations is $A = \{x \in X_i : \sum_{i \in N} x_i \leq \bar{\omega}\}$.

imposes that agent i receives the same bundle both at t and t' . The reason is that he cannot distinguish between these two states based on the observed equilibrium prices.

¹¹We will order vectors with the usual conventions $\gg, >, \geq$.

Let $P = \mathbb{R}_{++}^l$ be the set of strictly positive *price vectors*. For each $i \in N$, let $B_i(p) = \{x_i \in X_i : p \cdot x_i \leq p \cdot \omega_i\}$ be i 's *budget set* at a given price p . Similarly, let $B_i(p)|_{x_i \leq \bar{\omega}} = \{x_i \leq \bar{\omega} : p \cdot x_i \leq p \cdot \omega_i\}$ be i 's *constrained budget set* at p .

Definition 1: A *constrained Walrasian equilibrium* is a pair $(p^*, x^*) \in \mathbb{R}_+^l \times \mathbb{R}_+^{\ell n}$ such that $\sum_i x_i^* = \bar{\omega}$, and for each $i \in N$, $x_i^* \in \arg \max_{x_i \in B_i(p^*)|_{x_i \leq \bar{\omega}}} u_i$.

For each $E \in \mathcal{E}$, let $CWE(E)$ be the set of CWE allocations of E . The CWE $W : \mathcal{E} \rightarrow A$ associates to each economy $E \in \mathcal{E}$ its set of CWE allocations $CWE(E)$.

A normal game form or *mechanism* is defined as $\Gamma = (M, g)$, where $M = \prod_i M_i$ is the message space and $g : M \rightarrow A$, is an outcome function that associates a feasible allocation with each path of play. Fixing the game (Γ, E) , the payoff that each player $i \in N$ receives after m being played is $u_i(g(m)^i)$. A *Nash equilibrium* of (Γ, E) is a strategy profile $m^* \in M$ such that for every $i \in N$

$$u_i(g(m^*)^i) \geq u_i(g(m'_i, m_{-i}^*)^i) \text{ for each } m'_i \in M_i.$$

The *set of Nash equilibrium outcomes* of (Γ, E) is denoted $NE(\Gamma, E)$. A normal game form Γ *implements* in Nash equilibrium the CWE correspondence if $NE(\Gamma, E) = CW(E)$ for each $E \in \mathcal{E}$.

We are now ready to define the mechanism we use.

The mechanism Γ :

Agents simultaneously announce the triple¹² $(x, p, n)^i \in A \times P \times \mathbb{N}$.

The outcome function is described as follows:

Rule 1: If for each $i \in N$, $(x, p)^i = (\bar{x}, \bar{p})$, $\bar{p} \cdot \bar{x}_i = \bar{p} \cdot \omega_i \forall i \in N$ and $\sum_i \bar{x}_i = \bar{\omega}$, then \bar{x} is implemented.

Rule 2: If $(x, p)^j = (\bar{x}, \bar{p}) \forall j \neq i^*$ —where $i^* = \min \{i \in N : n^i \geq n^j \forall j \in N\}$ —and if (\bar{x}, \bar{p}) satisfies the conditions presented in rule 1, $p^{i^*} = \bar{p}$, $x^{i^*} \neq \bar{x}$, $\bar{p} \cdot x^{i^*} = \bar{p} \cdot \omega_{i^*}$ and $x^{i^*} \leq \bar{\omega}$; then agent i^* gets x^{i^*} , the others $j \neq i^*$ get 0.

Rule 3: For all other cases, everybody receives their endowment, except agent $k^* = \max \{k \in N : n^k \leq n^j \forall j \in N\}$ who receives 0.

¹²Where \mathbb{N} is the set of positive integer.

Proposition 1: *The mechanism Γ implements the Constrained Walrasian correspondence in Nash Equilibrium in the class of economies \mathcal{E} .*

A proof for this proposition can be found in Bochet (2007). Since it relies on arguments standard in implementation theory, we omit it.

3 Incomplete Information: REE as an extension of WE?

3.1 Extending the Model

The structure is as in the previous section. Incomplete information is captured by the use of types. For each $i \in N$, let T_i be the finite set of *types* of agent i . $T_{-i} = \prod_{j \neq i} T_j$ is the Cartesian product of the set of types of players other than player i . Define $T = \prod_i T_i$ to be the set of possible *type profiles*. A *state of the world* is a collection of types $t = (t_1, t_2, \dots, t_n)$. A state summarizes agent's preferences and information. Thus preferences may vary across states and may be state dependent. We denote by $T^* \subseteq T$ the set of states occurring with positive probability. Each agent has a prior probability distribution on states of the world q_i , defined on T . Obviously, agents agree on 0 probability states, i.e. if $q_i(t) = 0$ for some $i \in N$, then $q_j(t) = 0 \forall j \neq i$. The set of all such states is $T \setminus T^*$. We make the following assumption on information structures.

Assumption 1: Information is non-exclusive (NEI)

For each $t \in T^*$, there is no $t'_i \in T_i \setminus \{t_i\}$ such that $t' = (t_1, \dots, t_{i-1}, t'_i, t_{i+1}, \dots, t_n) \in T^*$.

Assumption 2: No redundant type

For each $i \in N$ and each $t_i \in T_i$, there exists $t_{-i} \in T_{-i}$ such that $q_i(t_i, t_{-i}) > 0$.

Consumptions sets and individual endowments are state-independent and known to the planner. Hence, the set of feasible allocation is constant across states. In the class of economies \mathcal{E} , only the utility functions and the type of the consumers vary. In \mathcal{E} , the Bernoulli utility function of each agent $i \in N$ in each state $t \in T$, $u_i(\cdot, t)$, continuous, strictly increasing and quasi-concave.

More formally, $u_i : \mathbb{R}_+^l \times T \rightarrow \mathbb{R}_+$. An *economy with differential information* is defined as $E = \{(u_i, q_i, T_i)_{i \in N}\}$.

Definition 2: The environment is of *private values* if every agent is informationally autonomous. More formally, for each $i \in N$ and each $t \in T$, $u_i(\cdot, t) = u_i(\cdot, t_i)$.

Since information is non-exclusive, the mechanism can determine outcomes when the state reported is $s \notin T^*$. In the mechanism, agents will be restricted to announce allocations over states occurring with positive probability. A (feasible) state-contingent allocation (over states occurring with positive probability) $x : T^* \rightarrow A$ is a list of allocation, one for each state $t \in T^*$, where $x = (x_1, x_2, \dots, x_n)$ such that $\sum x_i(t) \leq \bar{\omega}$ for each $t \in T^*$. Denote by \mathbf{A} the set of (ex-post) feasible state-contingent allocations defined over T^* ,

$$\mathbf{A} = \{x : T^* \rightarrow A\}$$

Define by \mathbf{P} the set of state-contingent strictly positive price vectors – defined over states occurring with positive probability –, with element $p = (p_t, p_{t'}, \dots)$. For every $i \in N$, denote by $B_i(p_t) \subseteq X_i$ the budget set of agent i , at a given price p_t . Formally, $B_i(p_t) = \{x_i \in X_i : p_t \cdot x_i \leq p_t \cdot \omega_i\}$

Definition 3:

A *constrained rational expectation Equilibrium* (defined over states $t \in T^*$) is a pair (p^*, x^*) such that:

1) $p_t^* \cdot x_i^*(t) \leq p_t^* \cdot \omega_i$ for each $t \in T^*$ and each $i \in N$.

2) *Measurability of allocations with respect to prices:* For each agent i and each $t_i \in T_i$, $p_{(t_{-i}, t_i)}^* = p_{(t'_{-i}, t_i)}^* \implies x_i^*(t_{-i}, t_i) = x_i^*(t'_{-i}, t_i)$.

3) $\sum_{t_{-i} \in T_{-i}} q_i(t_{-i}|t_i) u_i(x_i^*(t_{-i}, t_i), t_i) \geq \sum_{t_{-i} \in T_{-i}} q_i(t_{-i}|t_i) u_i(y_i(t_{-i}, t_i), t_i)$, for each $i \in N$, each $t_i \in T_i$ and for each measurable y_i such that $y_i(t) \in B_i(p_i^*)|_{y_i(t) \leq \bar{\omega}}$ for each $t \in T^*$.

$$4) \sum_{i \in N} x_i^*(t) = \bar{\omega} \text{ for each } t \in T^*.$$

Definition 4: A *deception* for agent i is a mapping $\alpha_i : T_i \rightarrow T_i$. The interpretation is that when agent i is of type t_i , he acts (or report) as if he was of type $\alpha_i(t_i)$. Notice that by the definition of α_i , it is possible that $\alpha_i(t_i) = t_i$, i.e. agent i reports *truthfully* her type. Truthtelling is just the identity mapping and is denoted $\hat{\alpha}$. We denote by $\alpha(t) = (\alpha_1(t_1), \dots, \alpha_n(t_n))$ a collection of deception strategies. Thus, a profile of deception generates the mapping $\alpha : T \rightarrow T$. Define $x_\alpha = x \circ \alpha = (x(\alpha(t)), x(\alpha(t')), \dots)$; that is, $x \circ \alpha(t) = x(\alpha(t))$.

Definition 5: Define by $\mathbb{F} = \{x : T \rightarrow A\}$ the set of all social choice functions. A *social choice set* F is a subset of \mathbb{F} . Since information is non-exclusive and the mechanism defines outcomes in the event that an incompatible report of a state $s \notin T^*$ occurs, we restrict attention to $\mathbb{F}^* = \{x : T^* \rightarrow A\}$. In that case $\mathbb{F}^* = \mathbf{A}$.

Definition 6: A social choice correspondence $\mathcal{F} : \mathcal{E} \rightarrow 2^X$ defined on the domain \mathcal{E} is a set-valued function which assigns to every economy E in \mathcal{E} a social choice set F . \mathcal{F} is said to be *globally implementable in Bayesian equilibrium* relative to \mathcal{E} if, for all $E \in \mathcal{E}$, $\mathcal{F}(E)$ is *Bayesian implementable*. Again, we restrict attention to $\mathcal{F}^* : \mathcal{E} \rightarrow 2^{X^*}$ because of NEI.

The play of the game takes place at the interim stage. Agents know their own type. The interim expected utility of each agent $i \in N$, when of type $t_i \in T_i$, is

$$U_i(\cdot | t_i) = \sum_{t_{-i} \in T_{-i}} q_i(t_{-i} | t_i) u_i(\cdot, (t_{-i}, t_i))$$

A mechanism, is still an array $\Gamma = (M, g)$. The difference with the previous section is that each agent $i \in N$ chooses messages m_i as a function of his types. We call a mapping $\sigma_i : T_i \rightarrow M_i$ a *strategy* for agent i and Σ_i his set of strategies. Given a *strategy profile* $\sigma = (\sigma_1, \dots, \sigma_n)$ with $\sigma(t) = (\sigma_1(t_1), \dots, \sigma_n(t_n))$, $g(\sigma) = \{g(\sigma(t), g(\sigma(t')), \dots)\}$ represents the allocation rule which results when σ is played. Suppose the strategy profile σ is played. Let $g(\sigma)^i$ stand for bundles obtained by consumer i at the allocations prescribed by the path induced by σ , that is, $g(\sigma)$. Fixing the game (Γ, E) , the payoff that each player $i \in N$ receives after σ being played is (ex-post), in state t , $u_i(g(\sigma(t))^i, t)$.

A *Bayesian equilibrium* of a game with incomplete information is a profile of strategies $((\sigma_i(t_i))_{i \in N, t_i \in T_i})$, such that, for each $i \in N$, each $t_i \in T_i$,

$$U_i(g(\sigma)^i | t_i) \geq U_i(g(\sigma'_i, \sigma_{-i})^i | t_i) \text{ for each } \sigma'_i \in \Sigma_i.$$

The set of *Bayesian equilibrium outcomes* of (Γ, E) is denoted $BE(\Gamma, E)$. The set of CREE allocations of a given economy E is denoted $CREE(E)$. A mechanism Γ *globally implements in Bayesian equilibrium* the CREE correspondence if

$$BE(\Gamma, E) = CREE(E) \text{ for each } E \in \mathcal{E}.$$

3.2 Extending the mechanism: achievability of CREE

The message space of agents is enlarged to include report of private information, state-contingent allocations and prices for states occurring with positive probability. Since the environment satisfies NEI, incentive compatibility is indeed still a necessary condition but is vacuously satisfied. To see this, notice that we have to pay attention to the collection of types that are reported. Since $T^* \subset T$, it is possible that the collective report of types is incompatible. An obvious feature of NEI environment is that if $t \in T^*$ has been reported, then any unilateral deviation in type by an agent would result in an incompatible report $t' \notin T^*$. Since incompatible reports can be detected, incentive compatibility is easily satisfied. For instance, in case of an incompatible report, the planner may confiscate the aggregate endowment.

Before proceeding to the mechanism, let us recall that CREE allocations may not be ex-post individually rational, state-by-state- Walrasian or interim efficient. However, in private values environments, every CREE allocation is ex-post individually rational and state-by-state Walrasian.

The mechanism Γ :

Agents simultaneously announce the quadruple $(x, p, n, t_i)^i \in \mathbf{A} \times \mathbf{P} \times \mathbb{N} \times T_i$.

The outcome function is described as follows:

Rule 1: If the collection of types reported s is compatible¹³, $(x, p)^i = (\bar{x}, \bar{p}) \forall i \in N$, $\bar{p}_t \cdot \bar{x}_i(t) = \bar{p}_i \cdot \omega_i \forall i \in N, \forall t \in T^*$ and $\sum_i \bar{x}_i(t) = \bar{\omega} \forall t \in T^*$, then $\bar{x}(s)$ is implemented.

¹³A report of types $(s_i)_{i \in N}$ is said to be compatible if $s \in T^*$.

Rule 2: If the collection of types s reported is compatible, if $(x, p)^j = (\bar{x}, \bar{p}) \forall j \neq i^*$ where $i^* = \min \{i \in N : n^i \geq n^j \forall j \in N\}$ and if (\bar{x}, \bar{p}) satisfies the conditions presented in rule 1, $p^{i^*} = \bar{p}$, $x^{i^*} \neq \bar{x}$, $\bar{p}_t \cdot x_{i^*}^{i^*}(t) = \bar{p}_t \cdot \omega_{i^*} \forall t \in T^*$ and $x_{i^*}^{i^*}(t) \leq \bar{\omega} \forall t \in T^*$; then agent i^* gets $x_{i^*}^{i^*}(s)$, agents $j \neq i^*$ get 0 (and the rest of the goods is thrown away).

Rule 3: For all other cases, everybody receives their endowment, except agent $k^* = \max \{k \in N : n^k \leq n^j \forall j \in N\}$ who receives 0.

We characterize the set of equilibrium outcomes. It is a natural extension of the set of equilibrium outcomes obtained for the complete information case. Suppose σ is a Bayesian Equilibrium such that for each $i \in N$ and each $t_i \in T_i$, $\sigma_i(t_i) = (x, p, n, \alpha_i(t_i))^i$, and $g(\sigma) = a$.

Proposition 2: *Extension of the Walrasian correspondence:*

- 1) *Every equilibrium outcome is given by rule 1.*
- 2) *For every compatible deception α ($\alpha = \hat{\alpha}$ or $\alpha \neq \hat{\alpha}$), for every agent, \bar{x}_α is expected-utility maximizing over the Constrained budget sets generated by \bar{p}_α and individual endowments.*
- 3) *Moreover, when the environment is of private values, \bar{x}_α is state-by-state Walrasian*

Proof: The proof of the proposition is divided in a series of lemma. Lemma 4 and 5 prove the first part of the proposition. Lemma 6 and 7 prove respectively the second and third part of the proposition.

Lemma 4: *For every α ($\alpha = \hat{\alpha}$ or $\alpha \neq \hat{\alpha}$), there does not exist $t \in T^*$ such that $a(\alpha(t))$ comes from rule 3*

Proof: Suppose not. The deception α has been used, and there exists $t \in T^*$ for which the outcome $a(\alpha(t))$ is given by rule 3. Agent k^* , of type t_{k^*} receives 0 following the report $\alpha(t)$. Agent k^* can become agent i^* by appropriately announcing a different integer, and this with probability one, for every $\alpha(s)$ for which $\alpha_{k^*}(s_{k^*}) = \alpha_{k^*}(t_{k^*})$. Call him agent i^* , of type t_{i^*} , following the deviation. For $\alpha(t)$ reported, if the outcome still falls in rule 3 following the change in integer, agent i^* receives ω_{i^*} . Since $\omega_i > 0 \forall i \in N$ and preferences are strongly monotonic, this is profitable when $\alpha(t)$ is reported. On the other hand, if the outcome now falls in rule 2 when $\alpha(t)$ is reported, this deviator is awarded $x_{i^*}^{i^*}$. Since $p_{\alpha(t)} \gg 0$ and $\omega_{i^*} > 0$, it is the case that $x_{i^*}^{i^*} > 0$. By strong monotonicity of preferences, this is profitable when $\alpha(t)$ is reported. Now, for other $\alpha(s) \neq \alpha(t)$ with $\alpha_{i^*}(s_{i^*}) = \alpha_{i^*}(t_{i^*})$,

if $a(\alpha(s))$ was given by rule 2, notice that agent k^* was receiving 0 unless he was agent i^* when $\alpha(s)$ was reported. In that case, $a(\alpha(s))$ is now given by rule 3 following the change in integer announced. Again this is profitable when $\alpha(s)$ is reported. Otherwise, if for $\alpha(s)$, agent k^* was in fact already agent i^* , then the change in integer does not affect the outcome $a(\alpha(s))$. Finally, if $a(\alpha(s))$ was given by rule 1, notice that a change in integer does not affect the outcome in such a case. Therefore, this is a deviation that is interim profitable. A contradiction with m being a Bayesian Equilibrium.

A consequence of this lemma is that type-reports should always be compatible.

Lemma 5: *For every compatible α ($\alpha = \hat{\alpha}$ or $\alpha \neq \hat{\alpha}$), there does not exist $t \in T^*$ such that $a(\alpha(t))$ comes from rule 2*

Proof: Suppose not. The compatible α has been used and for $\alpha(t)$ reported, $a(\alpha(t))$ is given by rule 2. Agent $j \neq i^*$, of type t_j , receive the 0 bundle when state $\alpha(t)$ has been reported. By modifying her integer announced, she can become agent i^* , the agent with the highest integer, with probability one, for any $\alpha(s)$ with $\alpha_j(s_j) = \alpha_j(t_j)$. So, call her agent i^* , of type t_{i^*} . Thus, given the rules of the game, the outcome is now given by rule 3 with probability one when $\alpha(t)$ is reported. Therefore, given $\alpha(t)$, agent i^* would receive ω_{i^*} which dominates 0, since $\omega_{i^*} > 0$ and preferences are strongly monotonic. For the other $\alpha(s) \neq \alpha(t)$ for which $\alpha_{i^*}(s_{i^*}) = \alpha_{i^*}(t_{i^*})$, if the outcome was initially given by rule 2, it is now either given by rule 3 –thus profitable– or still given by rule 2 if the new agent i^* was already i^* for the report $\alpha(s)$ –and thus the outcome $a(\alpha(s))$ would remain unchanged–; or the outcome was given by rule 1, in which case, the outcome is unaffected by a change in the integer announced. Therefore, this deviation is interim profitable. A contradiction.

A consequence of this lemma is that equilibrium outcomes are always given by rule 1. In equilibrium, agents always agree on a state contingent price-allocation pair.

Lemma 6: *For every compatible α ($\alpha = \hat{\alpha}$ or $\alpha \neq \hat{\alpha}$), $\forall i \in N, \forall t_i \in T_i$, $\bar{x}_\alpha = \bar{x} \circ \alpha$ is maximal¹⁴ in the budget sets generated by $\bar{p} \circ \alpha$ and ω_i , and the feasibility constraints*

¹⁴That is, $\forall i, \forall t_i \in T_i, \bar{x}_\alpha \in \arg \max_{x_i} U_i(x|t_i)$ subject to $x_i(t) \leq \bar{\omega} \forall t \in T$, and the budget sets generated by \bar{p}_α and individual endowments.

Proof: Suppose not. The compatible α is used ($\alpha = \hat{\alpha}$ or $\alpha \neq \hat{\alpha}$) and there exists an agent i , of type t_i , with report $\alpha_i(t_i)$, an allocation $y \neq \bar{x}$, with $y_i(s) \leq \bar{\omega} \forall s \in T^*$, $\bar{p}_s \cdot y_i(s) = \bar{p}_s \cdot \omega_i \forall s \in T^*$, and such that,

$$U_i(y_\alpha|t_i) > U_i(\bar{x}_\alpha|t_i).$$

Agent i , of type t_i has a profitable deviation. She can modify the integer she announced so as to become agent i^* , the agent with the highest integer, with probability one, and announce the state-contingent allocation $y \neq \bar{x}$. By the rules of the game, the outcome is given by rule 2. Therefore, given a collective report $\alpha(s)$, agent i^* is awarded $y_{i^*}(\alpha(s))$; and this for every report $\alpha(s)$ for which $\alpha_{i^*}(s_{i^*}) = \alpha_{i^*}(t_{i^*})$. Agent i , of type t_i , by playing this deviation, obtains the expected utility defined above. This is an interim profitable deviation. A contradiction.

Lemma 7: *If the environment is of private values, then for every compatible α (with $\alpha = \hat{\alpha}$ or $\alpha \neq \hat{\alpha}$), and for every $\alpha(t) \in T^*$, $\bar{x}(\alpha(t))$ is Constrained Walrasian at t , with price $\bar{p}_{\alpha(t)}$.*

Proof: By the previous lemma, we already know that \bar{x}_α is expected utility maximizing over the budgets sets generated by individual endowments and \bar{p}_α , and the feasibility constraints. Since the environment is of private values, for every agent i of type t_i , for every $\alpha(s)$ for which $\alpha_i(s_i) = \alpha_i(t_i)$, $\bar{x}_i(\alpha(s)) \in \arg \max u_i(x_i, t_i)$ subject to $B_i(\bar{p}_{\alpha(s)})|_{x_i \leq \bar{\omega}}$. Therefore, for every compatible α , $(\bar{x} \circ \alpha, \bar{p} \circ \alpha)$ is state-by-state Constrained Walrasian.

Q.E.D.

This lemma concludes the characterization of the set of equilibrium outcomes. The important part is the characterization for economies that do not satisfy private values. Nevertheless, we proceed first with a short summary of the result for the private values case, followed by the non-private values and the main result of the paper.

In private values environments, every equilibrium allocation rule is state-by-state constrained Walrasian. The mechanism globally implements the social choice correspondence for which, for every economy $E \in \mathcal{E}$, the social choice set is composed of state-by-state CWE allocation rules. Notice that the set of Constrained state-by-state WE allocations is in general a superset of the set of CREE allocations: the definition of CREE excludes non-measurable allocations. Observe, however, that measurability is very strong and not meaningful in private values settings. However, if utility functions

are strictly quasi-concave, The state-by-state Walrasian correspondence and the CREE correspondence then coincide: measurability of allocations with respect to prices is automatically obtained.¹⁵

For environments that do not satisfy private values, the allocation \bar{x}_α should be a maximal element in the budget sets generated by \bar{p}_α and individual endowments –subject to the feasibility constraints. In fact, observe that for each equilibrium in which agents are truthful, (\bar{x}, \bar{p}) is state-by-state Walrasian. Therefore, in equilibrium in which agents are truthful, non-fully revealing CREE cannot be equilibrium allocations unless they are state-by-state Walrasian. However, in non-private values settings, the set of CREE allocations is neither a subset, nor a superset of the set of state-by-state CWE allocations. We will show that although CREE allocations are the best measurable state-contingent allocations given the prices that prevail, they may not be the best state-contingent allocations that agents can obtain in Γ . It is well known (for instance, see Laffont (1985)) that fully revealing REE are ex-post efficient since every allocation is a WE in the respective associated full information economy. However, non-fully revealing REE may not be ex-post efficient and thus not even interim efficient.

Theorem 1: *There exists economies $E \in \mathcal{E}$ for which the set of Constrained REE is not contained in the set of Bayesian Equilibrium outcomes of (Γ, E)*

Proof: The proof uses a counterexample.

$N = \{1, 2, 3\}$. $T_1 = \{t_1, t'_1\}$, $T_2 = \{t_2\}$, and $T_3 = \{t_3, t'_3\}$. There are two states in $T^* = \{t, t'\}$ with $t = (t_1, t_2, t_3)$ and $t' = (t'_1, t_2, t'_3)$. For agent 2, $q_2(s) = \frac{1}{2}$, $s = t, t'$.

$$\begin{aligned} u_1(\cdot, t_1) &= \frac{1}{2} \log x_1 + \frac{1}{2} \log x_2 & u_1(\cdot, t'_1) &= \frac{1}{3} \log x_1 + \frac{2}{3} \log x_2 & \omega_1 &= (1, 1) \\ u_2(\cdot, t) &= \frac{1}{3} \log x_1 + \frac{2}{3} \log x_2 & u_2(\cdot, t') &= \frac{2}{3} \log x_1 + \frac{1}{3} \log x_2 & \omega_2 &= (1, 1) \\ u_3(\cdot, t_3) &= \frac{1}{3} \log x_1 + \frac{2}{3} \log x_2 & u_3(\cdot, t'_3) &= \frac{1}{2} \log x_1 + \frac{1}{2} \log x_2 & \omega_3 &= (1, 1) \end{aligned}$$

There exists a non-fully revealing CREE in this economy. It is given by the following price-allocation pair:

$$\begin{aligned} p_t &= p_{t'} = \left(1, \frac{5}{4}\right) \\ x(t) &= \left(\left(\frac{9}{8}, \frac{9}{10}\right); \left(\frac{9}{8}, \frac{9}{10}\right); \left(\frac{3}{4}, \frac{6}{5}\right)\right) \\ x(t') &= \left(\left(\frac{3}{4}, \frac{6}{5}\right); \left(\frac{9}{8}, \frac{9}{10}\right); \left(\frac{9}{8}, \frac{9}{10}\right)\right). \end{aligned}$$

¹⁵For instance if agent i is of type t_i both in state t and t' , $t \neq t'$, by private values $u_i(\cdot, t) = u_i(\cdot, t') = u_i(\cdot, t_i)$. If $p_t = p_{t'}$, by strict quasi-concavity, the bundles that maximize u_i over constrained budget sets defined by p_t and $p_{t'}$ are necessarily the same.

Since agents 1 and 3 receive different bundles across states, if x is an equilibrium outcome of the mechanism, agents have to be truthful and the equilibrium comes from rule 1. Agent 2 has a profitable deviation because the mechanism does not impose measurability restrictions on the allocations announced. Notice that allocation x is not state-by-state Walrasian allocation at price $p = (1, \frac{5}{4})$. The interim utility obtained from x by agent 2 is $U_2(x|t_2) \approx 0.002$. Suppose agent 2 deviates and announces the highest integer, so as to be agent i^* , and proposes the allocation,

$$\begin{aligned} x'(t) &= ((1.125, 0.9); (0.75, 1.2); (1.125, 0.9)) \\ x'(t') &= ((0.75, 1.2); (1.5, 0.6); (0.75, 1.2)). \end{aligned}$$

The allocation x' gives an interim utility of 0.02 to agent 2. By using this deviation, agent 2 triggers rule 2 and is awarded $x'_2(s)$ when $s = t, t'$ is reported. This is an interim profitable deviation for agent 2.

Q.E.D.

Measurability restrictions proper to CREE do not arise from the strategic behavior of agents. In fact, a non-fully revealing CREE allocation can typically arise as equilibrium only through the use of deceptions –unless it is state-by-state Walrasian. Moreover, if utility functions are strictly quasi-concave, then no non-fully revealing CREE that is not state-by-state Walrasian can be a Bayesian equilibrium outcome.

An explanation for the result is the following. At a REE, the only information that an agent obtains –in addition to his own private information– is the one contained in the equilibrium prices. On the other hand, in the mechanism we use, there is a higher degree of centralization: an uninformed agent may propose an allocation based on information that he does not have, even after observing the aggregate trade or the equilibrium prices. This makes sense here because information is non-exclusive: the mechanism can easily extract the information held by the other individuals.

As a consequence, in order to obtain achievability of the CREE correspondence, we need to impose strong restrictions on the behavior of agents: measurability restrictions on allocations proper to CREE should be incorporated in the outcome function. By doing so, the deviation constructed in the counterexample is no longer possible. We refer the reader to Bochet (2007) for a proof of this result.

3.3 Conclusion

Using an elementary mechanism, similar to the one constructed in DSV, we followed an approach that has been used in the recent literature on Shapley-Shubik strategic market games. Our was to investigate the game-theoretic connections between WE and REE using an elementary mechanism. By extending the mechanism we designed, we find that the properties of equilibrium allocation rules are very close to the description of CREE allocations. However, the measurability restrictions on allocations proper to REE do not emerge naturally from the behavior of agents: there exists economies for which the set of CREE allocations is not contained in the set of Bayesian equilibrium outcomes of the mechanism. The degree of centralization is such that the mechanism can easily extract information held by individuals. Without additional restrictions, agents can suggest allocations based on information that they cannot extract from prices, and the equivalence between WE and REE cannot be obtained. This result may cast some doubts on the connections between WE and REE beyond anonymous and fully decentralized market settings.

References

1. Blume L., Easley D., 1990. "Implementation of Walrasian Expectations Equilibria." *Journal of Economic Theory* **51**, 207-227.
2. Bochet O. 2007. "Switching from Complete to Incomplete Information." *Journal of Mathematical Economics*, **43**, 735-748.
3. Codognato G., Ghosal S., 2003. "Self-Fulfilling Mechanisms and Rational Expectations in Market with a Continuum of Traders." *Journal of Mathematical Economics*, **39**, 421-431.
4. Dubey P., Geanakoplos J., Shubik M., 1987. "The Revelation of Information in Strategic Market Games, a Critique of Rational Expectations Equilibrium." *Journal of Mathematical Economics* **16**, 105-137
5. Dutta B., Sen A., Vohra R., 1995. "Nash Implementation through Elementary Mechanisms in Economic Environments." *Review of Economic Design* **1**, 173-203.
6. Forges F., Minelli E., 1997. "Self-fulfilling mechanisms and Rational Expectations." *Games and Economic Behavior* **75**, 388-406
7. Gale D., 1987. "Limit Theorems for Market with Sequential Bargaining." *Journal of Economic Theory* **43**, 20-54
8. Gottardi P., Serrano R., 2005. "Market Power and Information Reve-

lation in Dynamic trading.” *Journal of the European Economic Association* **3**, 1279-1317

9. Hurwicz L., 1979. “Outcomes Functions Yielding Walrasian and Lindhal Allocations at Nash Equilibrium Points.” *Review of Economic Studies* **46**, 217-225.

10. Laffont J.J., 1985. “On the Welfare Analysis of Rational Expectations Equilibria with Asymmetric Information.” *Econometrica* **53**, 1-30

11. Maskin E., 1999. “Nash Equilibrium and Welfare Optimality.” *Review of Economic Studies* **66**, 23-38.

12. Palfrey T., Srivastava S., 1987. “On Bayesian Implementable Allocations.” *Review of Economic Studies* **54**, 193-208

13. Postlewaite A., Wettstein D., 1989. “Feasible and Continuous Implementation.” *Review of Economic Studies* **56**, 603-611.

14. Radner R., 1979. “Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices.” *Econometrica* **47**, 655-678.

15. Schmeidler D., 1980. “Walrasian Analysis via Strategic Outcome Functions.” *Econometrica* **48**, 1585-1593.

16. Serrano R., 2002. “Decentralized Information and the Walrasian Outcome: a Pairwise Meeting Market with Private Values.” *Journal of Mathematical Economics* **38**, 65-89

17. Wettstein D., 1990. “Continuous Implementation of Constrained Rational Expectations Equilibria.” *Journal of Economic Theory* **52**, 208-222.

18. Wolinsky A., 1990. “Information Revelation in a Market with Pairwise Meetings.” *Econometrica* **58**, 1-23.