Economics of Uncertainty and Information

Answer key, Homework 2

Exercise 1:

1) Agent 1 and 2 are perfectly informed, i.e. when they know their own type, they know the state.

2) An allocation x is incentive compatible if for each $i \in N$, each $t_i \in T_i$,

$$\sum_{t \in T} q_i\left(t|t_i\right) u_i(x(t), t) \geq \sum_{t \in T} q_i\left(t|t_i\right) u_i(x(t_i', t_{-i}), t) \ \forall t_i' \in T_i$$

That is, given that the $n-1$ agents other than i are reporting their types truthfully, agent i does not have an incentive to lie about his type.

In the problem at hand, observe that if $n-1$ agents report their types truthfully, then the state is revealed. For instance, consider state $t = (t_1, t_2, t_3, t_4)$. Once agents $1, 2$ and 3 have reported truthfully that their type is t_1, t_2 and t_3, it is known that the true state is t: the configuration of type $(t_1, t_2, t_3, \cdot)$ occurs only in state t.

An outside authority knows the structure of T and $T^\star$. Hence, if t_1, t_2 and t_3 are reported truthfully, agent 4 cannot lie anymore about his type: the lie will automatically be detected by the outside authority. Though this authority cannot infer who lied, it is easy to enforce incentive compatibility by just announcing –prior to the announcement of types– that if the type profile s that is reported is not in $T^\star$ –that is, someone lied– then everyone receives the 0 bundle.

This particular information structure is called **non-exclusive information**.

3) The no-trade allocation is obviously ex-post efficient and ex-post individually rational.

4) Consider the following allocation (this is similar to the case we saw in class),

$$\begin{aligned} y(t) &= (1, 1, 1, 1) \\ y\left(t'\right) &= (1, 1, 0.9, 1.1) \\ y\left(t''\right) &= (1, 1, 1.1, 0.9) \end{aligned}$$

When the states are t' or t'', the expected utility received by agent 3 and 4 are,

$$\begin{aligned} U_3(y_3, t_3') &= \frac{1}{2}\sqrt{0.9} + \frac{1}{2}\sqrt{4\,(1.1)} = 1.52315 \text{ and,} \\ U_4(y_4, t_4') &= \frac{1}{2}\sqrt{4(1.1)} + \frac{1}{2}\sqrt{0.9} = 1.52315 \end{aligned}$$

The expected utility agents 3 and 4 they get at the no-trade allocation is 1.

Allocation y interim dominates the no-trade allocation. Obviously, y is interim individually rational since the expected utility obtained from the initial endowments is the utility obtained from the no-trade allocation.

A consequence: risk sharing and insurance opportunities can arise even if the uninformed agents have the same conditional probabilities over states of the world (in the example we saw in class, agents 3 and 4 differed in their conditional probability assessments). Here, what creates the opportunities for interim improvements are the differences in the utility functions of agents 3 and 4.

However, observe that this allocation is **not** ex-post individually rational. Consider agent 3 of type t_3' and state t'.

$$u_3(y_3(t'), t') = \sqrt{0.9} < 1 = u_3(\omega_3, t')$$

5) Too difficult for you so forget it!

6) If we eliminate agent 1 from this problem, the information structure is no longer one of non-exclusive information. The discussion in 1) does not apply anymore. An outside authority cannot infer from the truthful report of $n-1$ agents what the state is.

Exercise 2:

1) Since utility functions are strictly increasing in every good, there exists –in each state– at least n (the number of agents) Pareto efficient allocation.

Consider state t. Consider giving the aggregate endowment $\bar{\omega}$ to agent 1 and nothing to agent 2,3,...,n. Clearly, this way of allocating the goods is Pareto efficient in state t since utility functions are strictly increasing in every goods. Since there are n agents, there are n ways to give the aggregate endowment, in each state.

The following allocation is ex-post efficient and incentive compatible, i.e. $\Delta_p \cap \Delta^\star \neq \emptyset$. For each $s \in T$

$$x(s) = (x_1(s), x_2(s), ..., x_n(s)) = (\bar{\omega}, 0, ..., 0)$$

The allocation is ex-post efficient since utilities are increasing in every good. Notice that since the allocation is constant across state of the world, it is trivially incentive compatible: lying about my type never changes what I receive.

A question that you may ask is the following: are all ex-post efficient allocations incentive-compatible? The answer is clearly no. Try to find at least one....

2) Consider the following example. $N = \{1, 2\}$, there are two goods y and z. The aggregate endowment is state independent and equal to $\bar{\omega} = (1, 1)$. There are two states $t = (t_1, t_2)$ and $t' = (t'_1, t_2)$. The utility functions are as follows:

$$\begin{aligned} u_1(\cdot, t) &= y_1 \\ u_1(\cdot, t') &= z_1 \\ u_2(\cdot, t) &= u_2(\cdot, t') = y_1 + z_1 \end{aligned}$$

You can check that the following allocation is ex-post efficient and incentive compatible:

$$\begin{aligned} x(t) &= (x_1(t), x_2(t)) = ((1, 0), (0, 1)) \\ x(t') &= (x_1(t'), x_2(t')) = ((0, 1), (1, 0)) \end{aligned}$$

Exercise 3:

Let A be the set of $(feasible)$ allocations.

1) To find the set of ex-post efficient allocations, it suffices to find the set of Pareto efficient allocations in each state of the world.

In state t, agent 1 only likes good z and agent 2 only likes good y. There is only one Pareto efficient allocation which is the top left corner of the Edgeworth box, i.e.

$$((0, 2), (2, 0))$$

In state t', agent 1 likes both goods while agent 2 still like only good y. Here, we have a set of Pareto efficient allocations. Let $PE(t')$ be that set. We obtain that,

$$PE(t') = \{x \in A : z_1 = 2,\, y_1 \in [0,2]\} \cup \{x \in A : \; y_2 = 2,\, z_1 \in [0,2]\}$$

Clearly, every ex-post efficient allocation is incentive compatible. Why? Observe that the only agent who can lie about his type is indeed agent 1. Agent 2 is uninformed and comes with only one type. At t, agent 1 of type t_1 receives the allocation that gives him the highest utility. So agent 1 will never have an incentive to pretend to be of type t_1'. Moreover, this allocation gives 0 utility to agent 1 of type t_1'. Now, any efficient allocation in state t' gives agent 1 either 0 utility (the second part of $PE(t')$) or positive utility (the first part of $PE(t')$). So again, agent 1 of type t_1' will not have an incentive to pretend to be of type t_1.

2) Here, the allocation is not ex-post efficient. Indeed, it is not incentive compatible either: agent 1 'of type t_1' will always pretend to be of type t_1 since he then receives the aggregate endowment.

Therefore, this allocation is not decentralizable: there does not exist an institution through which this allocation can be obtained. In particular, no market mechanism can generate this allocation as the result of some trade.