

# **When Supply Chain Management meets Social Choice**

## **Theory; Fair Allocation rules**

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### ***Abstract***

Allowing small new comers to enter the market and sell your products might open up new markets for companies. Unfortunately widely used allocation rules favour big companies, and some rules even directly favour specific companies. Allocation rules that do not favour big or specific companies should be Envy-free, Anonymous, Strategy-proof, and Pareto-optimal. Although most allocation rules fulfil some of the characteristics only one, relatively easy to implement, allocation rule, the Uniform (allocation) Rule, fulfils them all.

Keywords: Supply Chain, Allocation Rules, Uniform Rule, Fair Allocation

### ***Introduction***

The ability to allow small new comers to enter the market and sell your products might open up new markets for companies. Unfortunately widely used allocation rules favour big companies who can, for instance, take the financial risk associated with order inflation (actually receiving, and thus buying, more than your true preference), new comers might not have the financial means to take such a risk. Some allocation rules even directly favour specific companies, putting new comers

in a competitive disadvantage and breaching Article 81.1.d (ex Article 85) of the Treaty of Rome (European Union, 1957).

This paper only focuses on single-peaked preferences (“more is strictly preferred to less up to a point, and less to more beyond it” (Ching, 1994, p. 131)) and cases of strictly access demand.

## ***Literature Review***

Little has been written about the actual rules for allocation in supply chains, what is written about allocation rules, mostly focuses on allocation games, the decision by the agents of the height of their order (Cachon & Lariviere, 1999a, 1999b), supply chain literature does not seem to focus on the fairness of these allocation rules.

To ensure an allocation rule is fair we have to define some characteristics a fair allocation rule should possess.

An agent should not prefer the allocation to another agent over his own

*Axiom 1: A fair allocation should be Envy-free*

Although Envy-freeness is considered one of the most important characteristics of a fair allocation rule (Alkan, Demange, & Gale, 1991; Beviá, 1998), to be truly fair an allocation rule should possess more characteristics (characteristics of a social optimal rule overlap with that of a fair allocation rule (Ehlers, 2002)).

A fair allocation rule should be Anonymous (Sprumont, 1991), equals should be treated in an equal manner (Ching, 1994),

*Axiom 2: A good allocation rule is Anonymous*

An agent should not be able to improve his outcome by misrepresenting his preference (Klaus, Peters, & Storcken, 1998)

*Axiom 3: A good allocation rule is Strategy-proof*

There should be no other allocation in which all at least one agent is better off and all other agents not worse off than in the previous allocation, the allocation should be efficient (Ehlers, 2002)

*Axiom 4: A good allocation rule is Pareto-optimal*

Many allocation rules do exist with very different characteristics, a couple of published rules are look at below.

### **Proportional rule**

When the proportional rule is used an agent whose reported preference is  $x$  % of the total orders, receives  $x$  % of the capacity (Cachon & Lariviere, 1999b; Mallik & Harker, 2004).

## **Ordering rule**

The agents are ordered in one way or another, for instance the hierarchical rule (based on a numbering) (Klaus, Peters, & Storcken, 1998) and lexicographic allocation (based on the alphabetic order) (Cachon & Lariviere, 1999a)), the first agent is assigned goods according to his reported preference, then the second agent is assigned goods according to his reported preference and so on, till there is no more good to assign. The last agent to be assigned goods may not receive his reported preference but could receive what is left (if this is less than his reported preference).

## **Equal division rule**

The good, as the name of this allocation rule suggests, is divided equally over all agents regardless their reported preference. Although the agents could be better off by redistributing parts of the allotments among themselves (Thomson, 1994), this will not be considered here.

## **Equal distance rule**

In this allocation rule, also known as the Linear allocation rule (Cachon & Lariviere, 1999b; Mallik & Harker, 2004; Netessine & Rudi, 2006)., the difference between demand and supply is evenly distributed over all agents (Herrero & Villar, 1999), all agents will receive their preference minus the difference between demand and supply divided by the number of agents, when one or more agents will receive less than zero the remainder is divided over the remaining agents (Thomson, 1994).

## **Uniform rule**

In the uniform rule all agents all goods are distribute equally under all agents without crossing a reported preference peak (all agents are assigned as much goods as the smallest peak), then the remaining goods are distributed over the agents that are not satisfied, again not crossing a reported preference peak, this continues till all goods are distributed (Storcken, 2005). Some agents will be assigned their reported preference and others will receive less then their reported preference.

## ***Findings***

The consequences of using an unfair allocation rule vary, when, for instance, an allocation rule that is not Strategy-proof is chosen an agent wanting to receive a higher allocation will inflate his order so that he is allocated more. Even though the Supply Chain is believed to be “better of under an allocation mechanism that induces agents to inflate their orders” (Cachon & Lariviere, 1999a, p. 1104), better can not be strictly defined by profits. Since, if possible, an agent will receive his placed order, regardless of it being an inflated order or his true preference, new comers will be more careful when inflating their orders, since they might not be able to effort the cost associated with an allocation exceeding their preference.

When an allocation rule is used that is not Pareto-optimal, or same-sided (Klaus, Peters, & Storcken, 1997, 1998), all agents would at least be equally happy with another allocation and at least one agents would be better off (Beviá, 1998). One

problem associated with this is that some agents will reallocate goods among themselves, letting the supplier lose track of his distribution.

If the allocation rule is not anonymous it is, in Europe, in violation of Article 81.1.d (ex Article 85) of the Treaty of Rome (European Union, 1957), since a deliberate competitive disadvantage is created.

To see which of the, above mentioned, allocation rules fulfils the above mentioned Axioms, I will take a closer look at them.

### **Proportional rule**

While it is one of the most widely used allocation rules (Cachon & Lariviere, 1999b) it is not a fair rule, although it is Pareto-optimal and Anonymous it is not Envy-free and not Strategy-proof (Klaus, Peters, & Storcken, 1998), an agent wanting to receive a higher percentage of the capacity will inflate his order so it reflects a higher percentage of the total orders. Hence, the proportional rule can not be called a fair allocation rule.

### **Ordering rule**

Any allocation based on an ordering of agents (for instance the hierarchical rule (based on a numbering) (Klaus, Peters, & Storcken, 1998) and lexicographic allocation (based on the alphabetic order) (Cachon & Lariviere, 1999a)) is per definition not Anonymous, two agents with the same order size, but on different location in the ordering, can receive a different allocation, this allocation rule is also

not Envy-free, an agent receiving nothing can easily prefer the allocation to an other agent over his own. However, this allocation rule is Pareto-optimal and Strategy-proof (Klaus, Peters, & Storcken, 1998). If the ordering is purely based on order size, this allocation rule would be Anonymous (agents with an equal order would be treated in an equal manner, but some order inflation might take place (to much might be better than nothing at all), and thus the Strategy-proofness would be lost. The ordering rule can also not be called a fair allocation rule.

### **Equal division rule**

Since all agents will receive an equal share of the goods that are being allocated, this allocation rule is Strategy-proof, Anonymous and Envy-free, since this allocation rule is not same-sided (while one agent can receive more than his preference, an other can receive less than his preference) this allocation rule is not Pareto-optimal (Klaus, Peters, & Storcken, 1997, 1998) and thus is also not a fair allocation rule.

### **Equal distance rule**

When the difference between demand and supply is evenly distributed over the agents inflating the preference will lead to a bigger allocation, which makes the equal distance rule not Strategy-proof (Herrero & Villar, 1999). It is also not Envy-free, since some agents maybe assigned an allocation closer to the preference of another agent than that agents assigned allocation. This allocation rule is Anonymous (as it treats equals in an equal manner (Herrero & Villar, 1999, 2000)), and, since this allocation rule is same-sided (all agents will receive less than their preference), it is a Pareto-optimal rule. But it is not a fair allocation rule.

## Uniform rule

The uniform rule is Envy-free (Barberà & Jackson, 1997; Sprumont, 1991), since agents will be at least at the some distance from their reported preference as other agents. This allocation rule is Anonymous (Barberà & Jackson, 1997), since equal agents are treated in an equal manner. This allocation rule is Strategy-proof (Barberà & Jackson, 1997; Sprumont, 1991), since will not get an allocation closer to his real preference by inflating or deflating his reported preference. And this allocation rule is Pareto-optimal (Klaus, Peters, & Storcken, 1997, 1998), since it is same-sided (Ehlers, 2002), an agent will either receive his reported preference or less. This allocation rule for fills all requirements and is thus a fair allocation rule.

Table 1 Characteristics of the rules

	Envy-free	Anonymous	Strategy-proof	Pareto-optimal
Proportional rule		X		X
Ordering rule			X	X
Equal division rule	X	X	X	
Equal distance rule		X		X
Uniform rule	X	X	X	X

## Conclusion

We can only conclude that the Uniform allocation rule is the only rule that is truly fair, since it is Envy-free, Anonymous, Strategy-proof, and Pareto-optimal. Why then is it not used in Supply Chains? This is certainly not because it is hard to implement,

although the proportional rule, the ordering rule, and the equal division rule are somewhat simpler to implement, a simple algorithm can easily use the Uniform rule (Barberà & Jackson, 1997). The proportional rule is the most used allocation rule because it is the most intuitive allocation rule, most people will only come up with the proportional rule (Cachon & Lariviere, 1999b).

Although this paper addresses the fairness of allocation rules in Supply Chains it is not based on empirical or computer simulated research and thus only remains a theoretical work. An empirical research using supply chains that use different allocation rules and comparing their efficiency, growth rate etc might provide a better base for a plied to use fair allocation rules.

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