INTRODUCTION INTO THE INFORMATION SOCIETY (EBC2018)

THEORY PROBLEMS

1. Consider the lottery $p = (0.5 \times 0 \ ; \ 0.5 \times 20)$. Suppose that Ann is risk averse and she has to choose between the lottery p and a sure outcome of 9. Which of the two is she going to choose.

2. Consider a basketball game between Boston Celtics and LA Lakers. Suppose that a betting company offers the following odds: 1.8 for the Celtics and 2.1 for the Lakers. That is, if someone bets 10 Euros on the Celtics winning, and the prediction is correct, he/she will receive 18 Euros in total (8 Euros profit), and likewise if someone bets 10 Euros on the Lakers winning, and the prediction is correct, he/she will receive 21 Euros in total (11 Euros profit).

   2.1. Can both odds (the ones given to the Celtics and the ones given to the Lakers) be simultaneously fair?

   2.2. Suppose that Bob has bet 10 Euros to the Celtics and 10 Euros to Lakers. Is this a rational choice? Elaborate.

3. Ann owns a car insurance company and Bob is a potential customer. Assume that the probability that Bob is involved in an accident is commonly known and is equal to 1/3. For simplicity, we also assume that the damage caused by the possible accident is also commonly known and it is equal to 9 Euros. The car's current value is also equal to 9, implying that an accident would completely destroy the car. Bob can only buy a full insurance for the car. Bob is risk averse: his utility function is given by $u_B = \sqrt{w}$, where w denotes his wealth. Ann is also risk-averse: her utility function is given by $u_A(\pi) = \sqrt{20 + \pi}$, where π denotes her profit.

   3.1. Find the maximum price that Bob is willing to pay for the insurance.

   3.2. Is Ann going to sell him the insurance at this price?

4. Suppose that Ann is risk-loving and she has two choices a_1 and a_2. If she chooses a_1 she receives a (monetary) payoff of 4. If she chooses a_2 then a fair coin is tossed ("Heads" and "Tails" occur with equal probability: 1/2). If the outcome is "Heads" then Bob and Carol play the first normal form game depicted below (G_1). If the outcome is "Tails" then Bob and Carol play the second normal form game depicted below (G_2). Bob and Carol observe the realization

(G_1)	c_1	c_2	c_3
b_1	3,1	2,0	2,0
b_2	1,5	9,4	1,10
b_3	0,0	8,10	0,9

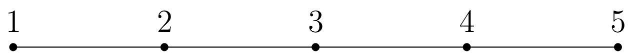
(G_2)	c_1	c_2	c_3
b_1	7,5	0,3	0,2
b_2	5,12	11,11	13,9
b_3	4,6	10,11	14,10

of the coin toss before they play, i.e., it is commonly known between them which game they play. Bob plays the row strategies b_i and Carol the column strategies c_j. The first number in every cell represents Bob's payoff and the second one represents Carol's payoff. Ann's payoff in every cell is equal to the average of the payoffs of Bob and Carol in that cell. Both Bob and Carol are risk-neutral. Furthermore, Bob and Carol are rational and it is commonly known between them that both are rational. What is Ann going to choose if she knows all the above?

5. Find the strategy profiles that survive iterated elimination of strictly dominated strategies in the following game.

	b_1	b_2
a_1	3,2	0,1
a_2	0,2	3,0
a_3	2,1	2,2

6. Consider a street with 5 houses as depicted in the picture below. The distance between any two neighbors is constant. Ann and Bob are potential store owners, and each of them has to choose where to locate his/her store on the street. The possible locations are exactly the points in $\{1, ..., 5\}$, i.e, for instance, none of them can open his/her store between 2 and 3. Note that it is allowed that both of them open their stores at the same place. Suppose that each house spends in total 10 Euros for goods they buy. Each house chooses the store that is closer. If the



two stores are equally close to a house, then the tenant of this particular house spends 5 Euros in each one of the stores. Find where Ann and Bob locate their stores if they are rational and this is common knowledge.

7. Consider two firms owned by Ann (the first one) and Bob (the second one), each having (constant) marginal cost equal to 2, and no fixed cost. Let the demand function be $Q = 10 - p$, and suppose that the two firms engage in Bertrand competition (they choose prices). The firm that charges the lowest price gains the entire market. If both firms charge equal prices, then they split the market.

   7.1. Suppose that the firms can charge any price in the interval $[0, 10]$. Find the (Nash) equilibria of this market.

   7.2. Suppose that they can charge only integer prices (whole numbers) from 1 to 5, i.e., $p \in \{1, ..., 5\}$. Find the (Nash) equilibria of this market.

8. Consider two firms owned by Ann (the first one) and Bob (the second one), each having (constant) marginal cost equal to 0, and no fixed cost. Let the (inverse) demand function be $p = 20 - Q$, and suppose that the two firms engage in Cournot competition (they choose quantities). Suppose for simplicity that they can supply only integer quantities (whole numbers) from 5 to 9, i.e., $q_i \in \{5, ..., 9\}$.

   8.1. What are the quantities supplied by the firms if they are rational and satisfy common knowledge of rationality?

   8.2. Find the quantities supplied by the two firms that bring the market to (Nash) equilibrium.

9. Ann is endowed with 2 Euros. At stage 1 she offers an integer amount to Bob (0 or 1 or 2). At stage 2 Bob either accepts or rejects Ann's proposal. If he accepts, he receives the amount that Ann has offered him and Ann receives the rest. If he rejects, they both receive 0.

   9.1. Find the Nash equilibria.

   9.2. Find the subgame perfect equilibria.

10. Ann and Bob participate in a first price sealed bid auction, with the special feature that if they place the same bid Bob wins the object. Ann's private value is $v_A = 6$ and Bob's private value is $v_B = 7$. Suppose they can bid any amount in the interval $[0, 10]$.

   10.1. Find the Nash equilibria.

   10.2. Are the equilibria you found in 10.1 also equilibria in the standard first price auction where ties are randomly broken (by tossing a coin that determines who wins the object)?

11. Ann and Bob participate in a second price sealed bid auction. Ann's private value is $v_A = 6$. Suppose they can bid any amount in the interval $[0, 10]$. Show that it is a weakly dominant strategy for Ann to bid her own private value.