Introduction into the information Society (EBC2018)

Applications

1. Consider two firms owned by Ann (the first one) and Bob (the second one), each having (constant) marginal cost equal to 0, and no fixed cost. Suppose that the two firms produce differentiated products, and let the demand function of the product of firm i be be $q_i = 10 - 2p_i + p_j$. Suppose that the two firms engage in Bertrand competition (they choose prices). Find the equilibrium prices.

2. Consider a market with demand function $Q = 10 - p$. There are two firms, an incumbent (owned by Ann) and a potential entrant (owned by Bob). They both have marginal costs equal to 2. The initial (sunk) cost for entering the market is equal to 6.

   2.1. Find the monopoly price charged by Ann before Bob entering the market.

   2.2. Suppose that Ann announces a price. If Bob enters they will both charge the announced price and they will share the market, and this is common knowledge between them. What price is Ann going to announce if she uses the limit pricing strategy? Is Bob going to enter the market if Ann charges this price?

3. Consider a market with two customers, Ann and Bob, with the following willingness to pay as a function of the quantity: $p_a = 10 - q_a$ and $p_b = 6 - \frac{1}{2}q_b$ respectively. Suppose that the (unique) firm serving this market has a marginal cost equal to 0.

   3.1. Assume that the firm charges one price to both customers. Find the price that maximizes the firms profit. Is this the monopoly price?

   3.2. Assume that the firm applies personalized pricing. Find the price the firm charges.

4. Consider a market with two customers, Ann and Bob, with the following willingness to pay as a function of the quality level: $p_a = 4 + Q$ and $p_b = 3$. Suppose that the firm offers at most 2 different versions (quality levels) of the product. Find the quality levels and the corresponding prices that maximize the firm's profits.

5. Consider a market with two customers, Ann and Bob, with the following willingness to pay as a function of the quality level: $p_a = 4 + Q$ and $p_b = 3 + Q$. Suppose that the firm offers at most 2 different versions (quality levels) of the product. Find the quality levels and the corresponding prices that maximize the firm's profits.

6. Consider a market with two customers, Ann and Bob, with the following willingness to pay as a function of the quality level: $p_a = 1 + 4Q$ and $p_b = 2 + Q$. Suppose that the firm offers at most 2 different versions (quality levels) of the product.

6.1. Find the quality levels and the corresponding prices that maximize the firm's profits.

6.2. Suppose that the customers have found a way to upgrade the low quality version to the high quality one without paying any cost. Find the quality levels and the corresponding prices that maximize the profits if the firm knows that the customers can upgrade.

7. Suppose a publisher (Ann) wants to sell to a customer (Bob) a Microeconomics book containing two parts: Theory and Applications. Each part may be either good or bad, and suppose that Bob's willingness to pay depends on the quality of the two parts, as illustrated in the following table:

	Good Applications	**Bad Applications**
Good Theory	40	30
Bad Theory	20	10

Now, suppose that Bob does not know the actual quality levels before having read the book, and therefore he does not know his own willingness to pay: He attaches equal probability, 1/4, to each of the four quality level combinations. On the other hand, Ann knows the actual quality levels, as well as Bob's willingness to pay for each quality level, and the corresponding probability that he attaches to each of them.

7.1. What price is Ann going to charge Bob?

7.2. Suppose that Ann can publish online (for free) one of the two parts of the book (she can decide which one). If the actual quality levels are Good Theory/Good Applications, is she going to publish online one of the parts? If yes, which one? How much is going to then charge for the book?

7.3. Suppose that Ann can publish online (for free) one of the two parts of the book (she can decide which one). If the actual quality levels are Good Theory/Bad Applications, is she going to publish online one of the parts? If yes, which one? How much is going to then charge for the book?

7.4. Suppose that Ann can publish online (for free) one of the two parts of the book (she can decide which one). If the actual quality levels are Bad Theory/Good Applications, is she going to publish online one of the parts? If yes, which one? How much is going to then charge for the book?

7.5. Suppose that Ann can publish online (for free) one of the two parts of the book (she can decide which one). If the actual quality levels are Bad Theory/Bad Applications, is she

going to publish online one of the parts? If yes, which one? How much is going to then charge for the book?

8. There are two customers, Ann and Bob, and one record company selling four different CD's. The willingness to pay that each customer has for each is depicted below:

	CD 1	**CD 2**	**CD 3**	**CD 4**
Ann	50	50	10	10
Bob	10	10	50	50

In order to listen to the CD's each customer needs to also buy a CD player, which costs 19.

8.1. How much is the record company going to charge for each CD?

8.2. Suppose that the record company can relax copyright protection, in which case Ann and Bob will be able to copy CD's from each other at no cost. How much is the record company going to charge for each CD then?

9. Suppose that there are two phone providers, Vodafone and T-Mobile, both with a marginal cost equal to 0. Suppose also that Ann does not have a contract with either of them and she has a willingness to pay of 4 per month. There is a 2 years horizon: Ann chooses a provider before $t = 1$, and before $t = 13$ she decides whether she wants to stick to the same provider or switch. The cost for setting up a new account is equal to 15, whereas her switching cost is equal to 15. Suppose that the companies can only charge integer prices per month, and if Ann is indifferent between the two offers she chooses T-Mobile.

9.1. Suppose that Ann has already chosen one of the two providers before $t = 1$. How much is this provider going to charge for the second 12-month period?

9.2. How much are they going to charge in equilibrium before $t = 1$?

9.3. Suppose that both providers can offer 12-month and 24-month contracts before $t = 1$. Contracts cannot be broken at any point before their expiration. Are they going to offer 12-month contracts, or 24-month contracts in equilibrium? How much are they going to charge in equilibrium?

9.4. Suppose that Ann does not already have a phone device. Buying one directly from Nokia costs 30. Both Vodafone and T-Mobile have an agreement with Nokia, according to which they can buy the phone at the price of 10. Suppose that both providers can offer both 12-month and 24-month contracts before $t = 1$. Contracts cannot be broken at any point before their expiration. Are they going to offer 12-month contracts, or 24-month contracts

in equilibrium? How much are they going to charge in equilibrium? Are they going to offer a phone for free together with this contract?

10. Suppose there are two providers for cell phone coverage, T-Mobile and Vodafone. Both of them have a constant marginal cost equal to 2. Suppose also that there are 10 customers with utility (from using a network) given by

$$u(p, n) = 10 + n^2 - p,$$

where p denotes the price per year and n the number customers in this network. T-Mobile has already 7 customers and Vodafone has 3 customers, and this is common knowledge. Every customer observes the price that each provider charges. Every customer believes that every other customer stays with the same provider and this is also common knowledge, and given these beliefs, he/she chooses the provider that gives him/her the highest utility (given the prices).

10.1. What prices are they going to charge in equilibrium?

10.2. Find the equilibrium prices if the two firms initially have 5 customers each.

11. Suppose there are two providers for cell phone coverage, T-Mobile and Vodafone. Both of them have a constant marginal cost equal to 2. Suppose also that there are 10 customers who are connected in a straight line, i.e., customer i is connected with $i - 1$ and $i + 1$ (everybody is connected with two people except 1 and 10 who are connected with only one person). Each customer's utility (from using a network) is given by

$$u(p, n) = 10 + n^2 - p,$$

where p denotes the price per year and n the number neighbors in this network. T-Mobile has already 7 customers and Vodafone has 3 customers, and this is common knowledge. Every customer observes the price that each provider charges. Every customer believes that all the neighbors stay with the same provider and this is also common knowledge, and given these beliefs, he/she chooses the provider that gives him/her the highest utility (given the prices).

11.1. What prices are they going to charge in equilibrium if 1,2 and 3 are with Vodafone and the rest are with T-Mobile?

11.2. What prices are they going to charge in equilibrium if 1,5 and 9 are with Vodafone and the rest are with T-Mobile?

12. Consider the population of customers, $C = \{c_1, ..., c_{1000}\}$. Assume that the customer $c_k \in C$ has a willingness to pay equal to $k \cdot n$, where n denotes the number of customers that buy the good. Derive the aggregate demand function.