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"New Solutions for the Minimal Cost Spanning Tree Problem"

Abstract:

We consider the familiar minimal cost spanning tree problem: n agents and a source live on $n+1$ distinct nodes; each agent needs to be connected to the source, either directly or indirectly through a path over any number of nodes. Given the cost of connecting each pair of nodes, the minimal cost graph is a tree rooted at the source, constructed by low complexity algorithms.

The minimal stability requirement of a set is cost shares is the Stand Alone Core property. A stronger requirement is Population Monotonicity (PM): adding a new user cannot increase the cost share of an existing agent.

Two other compelling requirements are Cost Monotonicity (CM), my cost share increases weakly in my connecting costs to other agents, and Continuity (CONT), cost shares are continuous in the matrix of connecting costs. Bergantinos and Vidal-Puga (BVP) proposed the first solution meeting PM, CM and CONT. Its computational complexity is low, because it is constructed from the irreducible cost matrix. Its drawback is that it ignores large amounts of relevant information about pairwise costs. In particular it fails Ranking (R): if the cost of connecting i to k is less than that of connecting j to k , for all k , i should pay less.

We introduce a linear extension technique, producing a reasonable solution once we know how to share costs on extreme points in the space of cost matrices. This delivers a family of solutions meeting PM, CM, CONT and R, as well as of low computational complexity. We also discuss plausible solutions in the Stand Alone Core violating Population Monotonicity.