

An Epistemic Course in Game Theory
Exercises to Lecture 4: “Proper rationalizability, An epistemic foundation for iterated weak dominance”

Problem 4.1: Proper rationalizability versus iterated weak dominance.

Consider the following game, which has also been discussed in the lecture.

		Opponent		
		<i>d</i>	<i>e</i>	<i>f</i>
You	<i>a</i>	2, 2	1, 1	0, 5
	<i>b</i>	1, 2	2, 1	0, 5
	<i>c</i>	0, 0	0, 6	0, 5

- (a) Use logical reasoning to find the properly rationalizable choices for you and your opponent in this game. Compare this to the choices selected by iterated weak dominance. Can you explain, intuitively, where this difference comes from?
- (b) Construct an epistemic model with lexicographic beliefs such that every type within this model is properly rationalizable. How many types do you need in total?
- (c) Let k be the number of steps needed in the algorithm of iterated weak dominance. Indicate how you can construct, within a *complete* epistemic model with lexicographic beliefs, a full support type t_1 for you that expresses k -th order assumption of rationality.

Problem 4.2: To which pub shall I go?

Suppose, you and Barbara will go out this evening. There are five pubs where you could go, and we call them a, b, c, d and e . Both you and Barbara don't know where the other person will be going. The preferences for you and Barbara are given by the following table:

	Pub a	Pub b	Pub c	Pub d	Pub e
You	1	1	1	1	1
Barbara	1	2	3	4	5

Barbara likes talking during the whole evening, but you prefer to be alone with your glass of beer. More precisely, if you both go to the same pub, this would decrease you utility by 1, and increase Barbara's utility by 2.

- (a) Formulate this situation as a static game between you and Barbara.
- (b) Which choice(s) can you make under iterated weak dominance?
- (c) By logical reasoning, determine the choice(s) you can make under proper rationalizability.
- (d) Construct an epistemic model with lexicographic beliefs such that
 - every type within this model is properly rationalizable, and
 - for every properly rationalizable choice found in (c) there is a type in this model that supports this choice.