**Corrections, improvements, additions to**

Hans Peters: *Game Theory – A Multi-Leveled Approach*, Springer, 2008, ISBN 978-3-540-69290-4

• Pages iv, viii: My email-address has changed to `H.Peters@maastrichtuniversity.nl`.

• Page 32, lines 2 and 5: a mixed strategy of player 1 is a probability distribution over the rows of $A$ and $B$, and a mixed strategy combination of player 2 is a probability distribution over the columns of $A$ and $B$.

• Page 52, line 17: replace $lL$ by $lR$ and $rL$ by $rR$; lines 27/28: replace $lL$ by $lR$ and $rL$ by $rR$; line 33: replace $lL$ by $lR$.

• Page 63, line 13: replace 'replay' by 'reply'.

• Page 66, last sentence: replace this sentence by 'But then type $t'$ of player 1 would gain by playing $L$ instead of $R$, so this cannot be an equilibrium.'

• Page 68, before last line: change to '(i.e., if the parts are not defective)'.

• Page 106, line 2: delete 'or'.

• Page 144, Table 10.2, caption: of course, $m_3$ and $w_3$ stay single.

• Page 171: change the formulation of Lemma 13.2 as follows:
*Let* $\mathbf{p}^* \in \Delta^m$ *and* $\mathbf{q}^* \in \Delta^n$. *Then* $\mathbf{p}^* \in \beta_1(\mathbf{q}^*)$ *if and only if* $\mathbf{p}^* A\mathbf{q}^* \geq \mathbf{e}^i A\mathbf{q}^*$ *for all* $i = 1, \ldots, m$ *and* $\mathbf{q}^* \in \beta_2(\mathbf{p})^*$ *if and only if* $\mathbf{p}^* B\mathbf{q}^* \geq \mathbf{p}^* B\mathbf{e}^j$ *for all* $j = 1, \ldots, n$.

• Page 171, in second part of Definition 13.4: change $\mathbf{p}A\mathbf{e}^j = \max_k \mathbf{p}A\mathbf{e}^k$ to $\mathbf{p}B\mathbf{e}^j = \max_k \mathbf{p}B\mathbf{e}^k$.

• Page 172, line 8, third term: change to $\sum_{k'=1}^{m} p_{k'} \max_k \mathbf{e}^k A\mathbf{q}$.

• Page 175, Figure 13.4: the set of Nash equilibria is, thus, $\{((1,0),\mathbf{q}) \mid q_2 = 0\} \cup \{(\mathbf{p},(1,0,0)) \mid 1 \geq p_1 \geq \frac{1}{2}\} \cup \{((\frac{1}{2},\frac{1}{2}),\mathbf{q}) \mid q_3 = 0\} \cup \{(\mathbf{p},(0,1,0)) \mid \frac{1}{2} \geq p_1 \geq 0\}$.

• Page 180/181. Replace the text following the proof of Lemma 13.14 up to the paragraph preceding Lemma 13.18 by the following text.

A perfect equilibrium is a strategy combination that is the limit of *some* sequence of Nash equilibria of perturbed games. Formally:

**Definition 13.15.** A strategy combination $\sigma$ is a *perfect equilibrium* if there is a sequence $G(\mu^t)$, $t \in \mathbb{N}$, of perturbed games with $\mu^t \to \mathbf{0}$ for $t \to \infty$ and a sequence of Nash equilibria $\sigma^t \in G(\mu^t)$ such that $\sigma^t \to \sigma$ for $t \to \infty$.

As follows from Theorem 13.17 below, a perfect equilibrium is a Nash equilibrium. So the expressions *perfect equilibrium* and *perfect Nash equilibrium* are equivalent.

Call a strategy combination $\sigma$ in $G$ *completely mixed* if $\sigma_i(h) > 0$ for all $i \in N$ and $h \in S_i$.

**Lemma 13.16.** *A completely mixed Nash equilibrium of $G$ is a perfect equilibrium.*

*Proof.* Problem 13.12. □

Also if a game has no completely mixed Nash equilibrium, it still has a perfect equilibrium.

**Theorem 13.17.** *Every finite game $G = (N, S_1, \ldots, S_n, u_1, \ldots, u_n)$ has a perfect equilibrium. Every perfect equilibrium is a Nash equilibrium.*

*Proof.* Take any sequence $(G(\mu^t))_{\mu^t \to \mathbf{0}}$ of perturbed games and $\sigma^t \in NE(G(\mu^t))$ for each $t \in \mathbb{N}$. Since $\prod_{i \in N} \Delta(S_i)$ is a compact set we may assume without loss of generality that the sequence $(\sigma^t)_{t \in \mathbb{N}}$ converges to some $\sigma \in \prod_{i \in N} \Delta(S_i)$. Then $\sigma$ is perfect. It is easy to verify that $\sigma \in NE(G)$. □

• Page 191, Problem 13.4, (b): $\mathbf{p}^*$ instead of $\mathbf{p}*$.

• Page 193, Problem 13.11: Show that $Y$ is never a best reply for player 3, and that $Y$ is not strictly (and not even weakly) dominated.

• Page 221, lines 2/3 of the proof of Prop. 15.15: the part 'for $i \notin C(\mathbf{x})$, $x_i = 0$ ' is redundant and can be omitted.

• Page 221, line 1 of the proof of Prop. 15.16: replace $i \notin C(\mathbf{x})$ by: $i \in N$.

• Page 221, line 5 of the proof of Prop. 15.16: the part 'whereas $x_i = 0$ ' is redundant and can be omitted.

• Page 222, before last line: $\varepsilon \in [0, 1]$.

• Replace the last part of the proof of Proposition 15.18, starting with 'The second term in the expression...' by

Note that the first term after the second inequality sign is positive. We will show that the second term is zero, which completes the proof of the proposition, since then $v(\mathbf{z}) > v(\mathbf{x})$. To show that $\sum_{i \notin C(\mathbf{x})} y_i \ln(z_i) = 0$, it is sufficient to show that $C(\mathbf{y}) \subseteq C(\mathbf{x})$. Suppose that $j \in C(\mathbf{y})$ and $j \notin C(\mathbf{x})$. By asymptotic stability of $\mathbf{x}$, $\xi(t, \mathbf{x}^0) \to \mathbf{x}$ for all $\mathbf{x}^0 \in U \cap \Delta_0^m$. Write

$$v(\xi(t,\mathbf{x}^0)) = \sum_{i:\xi_i(t,\mathbf{x}^0)>0, i\in C(\mathbf{x})} (y_i - x_i) \ln(\xi_i(t,\mathbf{x}^0)) + \sum_{i:\xi_i(t,\mathbf{x}^0)>0, i\notin C(\mathbf{x})} y_i \ln(\xi_i(t,\mathbf{x}^0)) \ .$$

The first term after the inequality sign is bounded by some constant $\gamma$ since $\xi_i(t,\mathbf{x}^0) \to x_i > 0$. The second term converges to $-\infty$ since $\xi_j(t,\mathbf{x}^0) \to x_j = 0$ and $y_j > 0$. But $v(\xi(t,\mathbf{x}^0)) \to -\infty$ contradicts the nondecreasingness of $v$ along the trajectory $\xi(t,\mathbf{x}^0)$. □

• Page 232. After the proof of Theorem 16.8 one may add:

Note that condition (16.1) is satisfied if the game $v$ has a non-empty core. So in that case, $C(v) = DC(v)$. See also Problem 16.11.

• Page 249, line 7 from below: see Problem 17.9 for the beta-integral formula.

• Page 261, line 3 from below:
Let $S \subseteq N$ be an arbitrary coalition, say $S = \{i_1, \ldots, i_s\}$ with $i_1 \leq \ldots \leq i_s$.

• Page 262, line 7:
This defines an ordering or permutation $\pi$, namely by $\pi(j) = i_j$ for all $j \in N$, with...

• Page 310. Extend Remark 22.2 to:

*Remark 22.2.* A consequence of Theorem 22.1 is that there are real numbers $\alpha$ and $\beta$ satisfying $\mathbf{y}\cdot\mathbf{z} > \alpha$ and $\mathbf{y}\cdot\mathbf{x} < \alpha$, and $\mathbf{y}\cdot\mathbf{z} > \beta$ and $\mathbf{y}\cdot\mathbf{x} = \beta$, for all $\mathbf{z} \in Z$ (notations as in the theorem). The last assertion is trivial. For the first assertion, note that in the proof of the theorem we have $\mathbf{y}\cdot\mathbf{z} \geq \mathbf{y}\cdot\mathbf{z}'$ for all $\mathbf{z} \in Z$, so $\mathbf{y}\cdot\mathbf{z}'$ is a lower bound for $\mathbf{y}\cdot\mathbf{z}$. Then take, for instance, $\alpha = \frac{1}{2}(\mathbf{y}\cdot\mathbf{z}' + \mathbf{y}\cdot\mathbf{x})$.

• Page 311, statement (1) of Lemma 22.4: $\mathbf{x} \geq \mathbf{0}$ instead of $\mathbf{x} > \mathbf{0}$.

• Page 312, line 12: ...the following two lemmas...

• Page 313, line 1: replace $A^t$ by $A^T$ in the $2 \times 3$-matrix. In line 4, replace $A^t$ by $A^T$ in the $2 \times 3$-matrix. (Superscript $T$ denotes the transpose.)

• Page 338. Replace second paragraph of the solution of Problem 12.6 by:

This is equivalent to existence of a vector $(\mathbf{q}, \mathbf{w}) \in \mathbb{R}^{n+m}$ with $\mathbf{q} \geq \mathbf{0}$, $\mathbf{w} \geq \mathbf{0}$, such that

$$\begin{pmatrix} A & I \\ \mathbf{e}^n & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{q} \\ \mathbf{w} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix} ,$$

where...

• Page 339, line 6: replace 'Nash equilibrium' by 'best reply'.

• Page 340, Problem 13.13, add:

(a) First observe that the set of Nash equilibria is $\{((p, 1-p), l, L) \mid 0 \leq p \leq 1\}$, where $p$ is the probability with which player 1 plays $U$.

• Page 343, Problem 15.9: In Case (1), the answer is $\mathbf{e}^2$.

• Due to an omission during the final production of the book, some of the page numbers in the Index may have shifted.