Additional corrections and improvements to

Hans Peters: *Game Theory – A Multi-Leveled Approach*, Springer, 2008, ISBN 978-3-540-69290-4

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- Page 56, Problem 4.8: replace the second bimatrix game by

$$\begin{array}{c} \\ T \\ B \end{array} \begin{array}{c} \begin{array}{cc} L & R \end{array} \\ \begin{pmatrix} 2,1 & 1,0 \\ 5,1 & 4,3 \end{pmatrix} \end{array} .$$

- Page 60, line 12: delete first 'type'.

- Page 61, caption of Fig. 5.1: add 'The upper numbers are the payoffs to player 1'.

- Page 61, line 9 from below: replace 'strategic' by 'extensive'.

- Page 61, lines 8–5 from below: replace the sentence starting with 'Also, every Nash equilibrium...' by the following paragraph:

Also, every Nash equilibrium is perfect Bayesian, since the only nontrivial information set (namely, that of player 1) is reached with positive probability (namely, equal to 1) for any strategy of player 2, and thus the beliefs are completely determined by player 2's strategy through Bayesian updating. More precisely, suppose the beliefs of player 1 are denoted by the nonnegative vector $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$, where $\alpha_1 + \ldots + \alpha_4 = 1$, from left to right in Fig. 5.1. For instance, α_3 is the probability (belief) attached by player 1 to player 2 having type n and playing strategy S. Suppose, for instance, that player 2 plays S if she has type y and B if she has type n, and let E denote the event that player 1's information set is reached. Then

$$\begin{aligned} \alpha_1 &= \text{Prob}\,[2 \text{ plays } S \text{ and has type } y \mid E] \\ &= \frac{\text{Prob}\,[2 \text{ plays } S, \text{ has type } y, \text{ and } E]}{\text{Prob}\,[E]} \\ &= \frac{\text{Prob}\,[2 \text{ plays } S \mid 2 \text{ has type } y]\,\text{Prob}\,[2 \text{ has type } y]\,\text{Prob}\,[E]}{\text{Prob}\,[E]} \\ &= 1 \cdot 0.5 = 0.5\ . \end{aligned}$$

In the same way we compute $\alpha_2 = 0$, $\alpha_3 = 0$, and $\alpha_4 = 0.5$. Similar computations can be made if player 2 would play mixed but we restrict attention here to pure strategies. What is important is that the beliefs of player 1 are always determined by computing the conditional probabilities since his (only) information set is always reached with positive probability, namely 1.

• Page 65, third line of last paragraph, after '...both equal to 1/2.' insert:

Formally, writing t for the event that 1's type is t and L for the event that player 1 plays L, we have:

$$\begin{aligned}\alpha &= \text{Prob}\,[t \mid L] = \frac{\text{Prob}\,[t \text{ and } L]}{\text{Prob}\,[L]} \\ &= \frac{\text{Prob}\,[L \mid t]\,\text{Prob}\,[t]}{\text{Prob}\,[L \mid t]\text{Prob}\,[t] + \text{Prob}\,[L \mid t']\text{Prob}\,[t']} \\ &= \frac{1 \cdot 1/2}{1 \cdot 1/2 + 1 \cdot 1/2} = 1/2\ .\end{aligned}$$

• Page 65, third paragraph, line 8, after '...probability 0;' insert:

formally, $\beta = \text{Prob}\,[t \mid R]$ and $\text{Prob}\,[t \mid R]\,\text{Prob}\,[R] = \text{Prob}\,[t \text{ and } R]$, but $\text{Prob}\,[R] = 0$, so that $\text{Prob}\,[t \mid R]$ is undetermined.

• Page 66, line 10: insert 'weakly' between 'possibly' and 'gain'.

• Page 80, at the end of Sect. 6.3, insert the following paragraph:

A few remarks on this equilibrium are in order. First, it is again Pareto inferior. E.g., both firms setting the monopoly price results in higher profits. Second, each firm plays a weakly dominated strategy: any price $c < p_i < a$ weakly dominates $p_i = c$, since it always results in a positive or zero profit whereas $p_i = c$ always results in zero profit. Third, the Bertrand equilibrium is beneficial from the point of view of the consumers: it maximizes consumer surplus.[1]

• Page 87, line 14: the number of rounds is $T + 1$.

• Page 87: replace the before last paragraph by:

We look for a subgame perfect equilibrium of this game, which can be found by backward induction. Note that subgames start at each decision node of a player, and that each node in Fig. 6.5 where a player has to accept or reject a proposal, actually represents infinitely many nodes, since there are infinitely many possible proposals leading to that node.

• Page 91, Problem 6.1: replace the last sentence by:

Compute the Nash equilibrium (assume that the parameters a, c_1, c_2 are such that the equilibrium amounts are indeed nonnegative).

[1]The occurrence of this equilibrium is often referred to as the *Bertrand paradox*.

• Page 92, Problem 6.7: replace by

6.7. *Variations on two-person Bertrand*

(a) Assume that the two firms in the Bertrand model of Sect. 6.3 have different marginal costs, say $c_1 < c_2 < a$. Derive the best reply functions and find the Nash-Bertrand equilibrium or equilibria, if any.

(b) Reconsider the questions in (a) for the case where prices and costs are restricted to integer values, i.e., $p_1, p_2, c_1, c_2 \in \{0, 1, 2, \ldots\}$. Consider the following cases: (i) $c_1 = c_2 = c < a - 1$ and (ii) $c_1 < c_2 - 1$, $a > 2c_2 - c_1$. (This reflects the assumption that there is a smallest monetary unit.)

• Page 170: replace line 7 by:

$$u_i(\sigma_1, \ldots, \sigma_n) = \sum_{(s_1,\ldots,s_n)\in S} \left(\prod_{j\in N} \sigma_j(s_j) \right) u_i(s_1, \ldots, s_n) \,.$$

• Page 154, line 5: add the condition '$a \neq b$' in the definition of Pareto Efficiency.

• Page 188, replace displayed formula in line 3 by

$$c_{(i,j)(h,k)} = \begin{cases} a_{ij} - a_{kj} & \text{if } i = h \in \{1, \ldots, m\} \text{ and } k \in \{1, \ldots, m\} \\ b_{ij} - b_{ik} & \text{if } j = h \in \{1, \ldots, n\} \text{ and } k \in \{1, \ldots, n\} \\ 0 & \text{otherwise.} \end{cases}$$

• Page 208, first line: replace the sentence starting with 'Denote...' by the following.

Spelled out completely, the argument is as follows. Denote the behavioral strategies of the players by $b_1(a)$ and $b_1(b) = 1 - b_1(a)$, $b_2(c)$ and $b_2(d) = 1 - b_2(c)$, and $b_3(e)$ and $b_3(f) = 1 - b_3(e)$. Suppose b_1, b_2, and b_3 are completely mixed. Let x denote the left node in player 2's information set h_2. Then $\mathbb{P}_b(x) = b_1(a)$ and $\mathbb{P}_b(h_2) = 1$, so by Bayesian consistency $\beta = b_1(a)$. Let y denote the left node in player 3's information set h_3. Then $\mathbb{P}_b(y) = b_1(a)b_2(c)$ and $\mathbb{P}_b(h_3) = b_2(c)$, so by Bayesian consistency $\beta' = b_1(a)b_2(c)/b_2(c) = b_1(a)$. Thus, $\beta = \beta'$ if the behavioral strategies are completely mixed. By consistency, $\beta = \beta'$ for all profiles of behavioral strategies.

• Page 235: replace Theorem 16.16 by:

Theorem 16.16. *Let (N, v) be a game. Then:*

(*a*) *The D-core of v is a subset of any stable set.*

(*b*) *If A and B are stable sets and $A \neq B$, then $A \not\subseteq B$.*

(*c*) *Suppose the D-core of v is a stable set. Then it is the unique stable set of the game.*

• Page 304, Problem 21.2: the zero-comprehensiveness condition is not needed and can be dropped.