

Alternating offers bargaining with loss aversion

Bram Driesen* Andrés Perea* Hans Peters*

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Abstract

The Rubinstein alternating offers bargaining game is reconsidered under the assumption that each player is loss averse and the associated reference point is equal to the highest turned down offer of the opponent in the past. This makes the payoffs and therefore potential equilibrium strategies dependent on the history of play. A subgame perfect equilibrium is constructed, in which the strategies depend on the history of play through the current reference points. It is shown that this equilibrium is unique under some assumptions that it shares with the equilibrium in the classical model: immediate acceptance of equilibrium offers, indifference between acceptance and rejection of such offers, and strategies depending only on the current reference points. It is also shown that in this equilibrium loss aversion is a disadvantage. Moreover, a relation with asymmetric Nash bargaining is established, where a player's bargaining power is negatively related to own loss aversion and positively to the opponent's loss aversion.

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1 Introduction

One of the characteristics of the Ståhl (1972) and Rubinstein (1982) non-cooperative approach to the classical problem of dividing a pie between two players is that preferences are time dependent but do not otherwise depend on the history of play of the game. In real life bargaining situations it is very likely that this assumption is violated and that the share of the pie that an agent finally obtains is evaluated in terms of the history of offers and counter-offers made so far. In particular, it is likely that a share of $x\%$ is evaluated less if a share of $y\%$ with $y > x$ has been within reach at an earlier stage of the game.

*Department of Quantitative Economics, University of Maastricht, P.O. Box 616, 6200 MD Maastricht, The Netherlands. Telephone: +31-43-3883835. Telefax: +31-43-3884874. Email addresses: B.Driesen@ke.unimaas.nl, A.Perea@ke.unimaas.nl, H.Peters@ke.unimaas.nl.

This is the phenomenon of *loss aversion*, and the objective of this paper is to study the effect of loss aversion in the Rubinstein alternating offers bargaining game.

Loss aversion was introduced by Kahneman and Tversky (1979). We adopt the simple and elegant version of Shalev (2002). In this version, an agent's preference is characterized by a *basic utility function*, a *reference point*, and a *loss aversion coefficient*: outcomes below the reference point are regarded as losses and their basic utility values are scaled down by the loss aversion factor. It is essential that this reference point be endogenous (see Kahneman and Tversky, 1984). In particular, in a game-theoretic context, what is regarded as a loss is likely to depend on the (history of play of the) game.

In the present paper we make the natural assumption that the reference point of a player is equal to the highest turned down offer of the opponent, since that represents the share of the pie that could have been obtained with certainty so far. It turns out that this results in a non-trivial modification of the Rubinstein alternating offers bargaining game: through changed reference points the game with loss aversion depends on the history of play. Indeed, while all subgames in the classical Rubinstein model starting with either a proposal by a player or an acceptance/rejection decision are essentially identical, in our case these subgames depend on the effect the history of play has on preferences. This makes the analysis of the game and, in particular, the characterization of subgame perfect equilibria, much more complicated.

Thus, we consider the Rubinstein alternating offers bargaining game with loss averse players, where the discount factor is interpreted as the probability of continuation of the game after rejection of a proposal. For this game, we construct a subgame perfect equilibrium that shares some features with the subgame perfect equilibrium of the classical game without loss aversion: immediate acceptance of equilibrium offers – implying that agreement is reached immediately as well; indifference between acceptance and rejection of such offers; and strategies depending only on the current reference points. It turns out that the equilibrium strategies depend on nine different regions describing the location of the players' reference points. For instance, if the reference points are high then we are essentially back in the classical case since higher offers are not going to be made in equilibrium and, thus, the reference points will no longer change. But if reference points are low – we assume them to be zero initially – then the equilibrium proposals are influenced by the possibility of future higher reference points. We also show that the constructed equilibrium is the unique subgame perfect equilibrium with the three mentioned properties, but have to leave it as an open problem whether these properties are necessary for uniqueness.

We include an extensive comparative statics analysis of the constructed subgame perfect equilibrium. We establish the intuitive result that higher loss aversion leads to a lower equilibrium share of the pie. We also extend our results to the case of different discount factors (probabilities of continuation of the game), and to more general (increasing and concave) basic utility functions. Finally, we establish an asymptotic relation with non-symmetric Nash bargaining solutions, in the spirit of Binmore *et al.* (1986). In particular, we show

that a player's bargaining power is negatively related to own loss aversion and positively to the opponent's loss aversion.

Further related literature

Loss aversion with a fixed reference point can be regarded as a special case of risk aversion. The effect of risk aversion in the Nash (1950) and Rubinstein (1982) bargaining models has been studied before, initially in Kihlstrom *et al.* (1981) and Roth (1985) in the context of expected utility. For the more general context of rank dependent utility, see Safra and Zilcha (1993) and Köbberling and Peters (2003). In all this work the utility functions are fixed and do not change as a result of (play of) the game.

Closer to the present context is Shalev (2002), who applies a loss aversion transformation to the discount factor, and obtains the unique subgame perfect equilibrium of Rubinstein with the transformed discount factors. Li (2007) assumes that bargainers prefer disagreement over any share that is below the highest they have been offered in the past, and finds a unique subgame perfect equilibrium in this setting. Thus, Li's way of updating reference points during play of the game bears some resemblance with our approach. However, Li assumes that reference points grow over time. Moreover, the utility functions in Li's model have a discontinuous jump that explicitly ensures that, in equilibrium, each offer made to a player exceeds the previous offer made to that player.

Compte and Jehiel (2003) assume that after breakdown of the negotiations, the game starts anew at a fixed cost. The first mover is randomly chosen from the two bargainers, and in each such new bargaining phase, they have a constant reference point which is based on the proposals made in the previous phases. The subgame perfect equilibrium they find in this setting has in common with Li's that bargainers do not achieve agreement instantaneously.

Organization of the paper

Section 2 describes the model and in Section 3 we construct a subgame perfect equilibrium. Section 4 concerns uniqueness of this equilibrium, and Section 5 collects our comparative statics results. Section 6 concludes. Because of their length, all proofs are collected in three Appendices A, B, and C.

2 The alternating offers bargaining model with loss aversion

In this section we describe the alternating offers bargaining model of Rubinstein (1982) and introduce the concept of loss aversion in this model. We will try to be as nontechnical as is possible without becoming imprecise. A completely formal treatment is presented in Appendix A.

One unit of a perfectly divisible good, the pie, has to be divided among two bargainers, 1 and 2. The set of all possible partitions of the pie is denoted as

$$Z := \{(z_1, z_2) \in \mathbb{R}^2 \mid z_1 + z_2 = 1, z_1, z_2 \geq 0\} .$$

Bargaining takes place at time $t = 1, 2, \dots$. At odd moments, player 1 makes a proposal $z = (z_1, z_2) \in Z$ and player 2 decides to accept (Y) or to reject (N) this proposal. At even moments, the roles of the players are reversed. If a proposal $(z_1, z_2) \in Z$ is accepted, then the game ends and each player i obtains z_i . If a proposal is rejected then the game continues to the next moment with probability $0 < \delta < 1$, and stops with probability $1 - \delta$. In the latter case, the game ends in disagreement and no player receives anything, i.e., the shares $(0, 0)$ result. If the game continues forever – which happens with probability 0 – then again the shares $(0, 0)$ result.

A strategy f for player 1 in this game specifies for each odd moment a proposal in Z , where this proposal may depend on the complete history of play of the game so far; and for each even moment an answer Y or N , where this answer may depend on the current proposal and on the rest of the history of play of the game. A strategy g of player 2 is defined similarly, with the roles of odd and even moments reversed.

So far, this is the Rubinstein alternating offers mechanism. From here, however, we deviate by assuming that the players are loss averse. We assume that the *basic utility* for a player i of obtaining a share z_i of the pie is just equal to z_i ,¹ but that shares z_i below some *reference point* $r_i \in [0, 1]$ are regarded as losses and scaled down by a *loss aversion coefficient* $\lambda_i \geq 0$. More precisely, player i evaluates z_i by the function

$$w(z_i, r_i, \lambda_i) := \begin{cases} z_i & \text{if } z_i \geq r_i , \\ z_i - \lambda_i(r_i - z_i) & \text{if } z_i < r_i . \end{cases}$$

or, equivalently, by

$$w(z_i, r_i, \lambda_i) = (1 + \lambda_i)z_i - \lambda_i \max\{r_i, z_i\} . \quad (1)$$

Loss aversion was first introduced by Kahneman and Tversky (1979). We use the simplified version of Shalev (2002). We assume that the loss aversion coefficients of the bargainers are given and fixed. The reference points of the players, however, are determined endogenously, in the following way.² At some moment t , consider all the offers made to player i by the other player j so far, possibly including the offer that is on the table at moment t . These represent all the shares of the pie that player i could have obtained up to this moment with certainty. Then, it is natural to assume that the maximum of those shares is player i 's reference point, since this is what he could have obtained: lower shares

¹This assumption will be relaxed in Section 5.

²Endogeneity of the reference point is an essential assumption in prospect theory, see Kahneman and Tversky (1984). If we would assume exogenous, fixed reference points then our model would reduce to a special case of the standard Rubinstein bargaining model.

represent losses with respect to this reference point, and are evaluated according to (1).

Thus, we assume that the initial reference points are 0 and write $r_1^0 = r_2^0 = 0$; and that at any moment $t \geq 1$, player 1's and 2's reference points are equal to

$$r_1^t = \max\{0, z_1^s \mid s = 2, 4, \dots \leq t\}, \quad r_2^t = \max\{z_2^s \mid s = 1, 3, \dots \leq t\},$$

if $z^1, z^2, \dots, z^t \in Z$ are the proposals made up to time t . Observe that incorporating loss aversion in this natural way causes a crucial difference with the classical Rubinstein bargaining game: the game is no longer history independent. For instance, subgames starting at odd moments are no longer identical copies of the whole game, since reference points and therefore payoff functions may have changed during the play of the game.

Recall that the unique subgame perfect equilibrium (SPE) in the classical Rubinstein bargaining game (without loss aversion) has the following characteristics: (i) it is time and history independent, that is, players always make and accept the same proposals; (ii) in equilibrium every proposal is immediately accepted; and (iii) in equilibrium a player is always indifferent between acceptance and rejection. Also in our model with loss aversion we look for an SPE but, clearly, we cannot expect to find a history independent equilibrium. In the next section, we will construct an SPE in which, instead, the strategies of the players are stationary Markov strategies: proposals and acceptance/rejection decisions depend only on the current reference points, and not on time or on the history of play of the game otherwise than through the effect on reference points. This equilibrium will still satisfy (ii) and (iii) and, in fact, we will show that it is the unique SPE with stationary Markov strategies and satisfying (ii) and (iii). Whether dropping one or more of these three conditions allows for different SPE is an open question.

3 A subgame perfect equilibrium

Heuristically, the SPE in the Rubinstein model is based on the idea that the proposal of a player i should make his opponent j indifferent between that proposal and the proposal of j himself in the next round. We will employ the same idea to construct an SPE in our model with loss averse players.

Consider an odd time moment t , where player 1 makes a proposal $x \in Z$. Suppose that after rejection player 2 makes the proposal $y \in Z$ at time $t + 1$. Let r_2 be player 2's reference point at time $t - 1$ (so $r_2 = x_2$ at $t = 1$). To make player 2 accept the proposal x we would need

$$(1 + \lambda_2)x_2 - \lambda_2 \max\{r_2, x_2\} \geq \delta [(1 + \lambda_2)y_2 - \lambda_2 \max\{y_2, \max\{r_2, x_2\}\}] - (1 - \delta)\lambda_2 \max\{r_2, x_2\} . \quad (2)$$

That is, player 2 should value the offer x at time t at least as highly as his own offer y in the next period *after having rejected* x . The analogous inequality at

even moments is

$$(1 + \lambda_1)y_1 - \lambda_1 \max\{r_1, y_1\} \geq \tag{3}$$

$$\delta [(1 + \lambda_1)x_1 - \lambda_1 \max\{x_1, \max\{r_1, y_1\}\}] - (1 - \delta)\lambda_1 \max\{r_1, y_1\} .$$

To construct the equilibrium, we assume that the inequalities (2) and (3) are equalities. Let $\mu_i := 1 + \lambda_i(1 - \delta)$ for $i = 1, 2$. Then elaborating (3) with equality yields the following three cases:

1. $r_1 > x_1 > y_1$: $\delta x_1 = y_1$.
2. $x_1 \geq r_1 > y_1$: $\delta x_1 = (1 + \lambda_1)y_1 - \delta\lambda_1 r_1$.
3. $x_1 > y_1 \geq r_1$: $\delta x_1 = \mu_1 y_1$.

These three cases are exhaustive – it is easy to check that the case $y_1 \geq x_1$ cannot occur. From (2) we obtain three similar cases:

- I. $r_2 > y_2 > x_2$: $\delta y_2 = x_2$.
- II. $y_2 \geq r_2 > x_2$: $\delta y_2 = (1 + \lambda_2)x_2 - \delta\lambda_2 r_2$.
- III. $y_2 > x_2 \geq r_2$: $\delta y_2 = \mu_2 x_2$.

By combining these equations we obtain a partition of the unit square $[0, 1]^2$ of all possible pairs of reference points (r_1, r_2) into nine sets. These sets are accurately depicted in Figure 1, and denoted by $X_{1,I}, \dots, X_{3,III}$. Within one and the same set X_{\cdot} , the equilibrium proposals of the players take the same form, which may or may not depend on the specific values of the reference points within the set.

A formal description of these nine sets and of all associated equilibrium proposals is given in Appendix A. Here we limit ourselves to the main aspects.

In the set $X_{1,I}$ the reference points are relatively high and the classical Rubinstein proposals

$$x^{1,I} = \left(\frac{1}{1 + \delta}, \frac{\delta}{1 + \delta} \right), \quad y^{1,I} = \left(\frac{\delta}{1 + \delta}, \frac{1}{1 + \delta} \right) .$$

obtain. These are independent of the values of the reference points.

Also in the sets $X_{1,III}$ and $X_{3,I}$ the equilibrium proposals are constant and do not depend on the reference points. In $X_{3,III}$, which is the relevant set for the equilibrium outcome in Theorem 3.1 below, the equilibrium proposals are

$$x^{3,III} = \left(\frac{\mu_1(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2}, \frac{\delta(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} \right), \quad y^{3,III} = \left(\frac{\delta(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2}, \frac{\mu_2(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} \right) .$$

Again, these proposals do not depend directly on the reference points.

In $X_{1,II}$ and $X_{3,II}$ the equilibrium proposals depend explicitly on player 2's reference point but not on player 1's reference point; and in $X_{2,I}$ and $X_{2,III}$ the equilibrium proposals depend explicitly on player 1's reference point but not

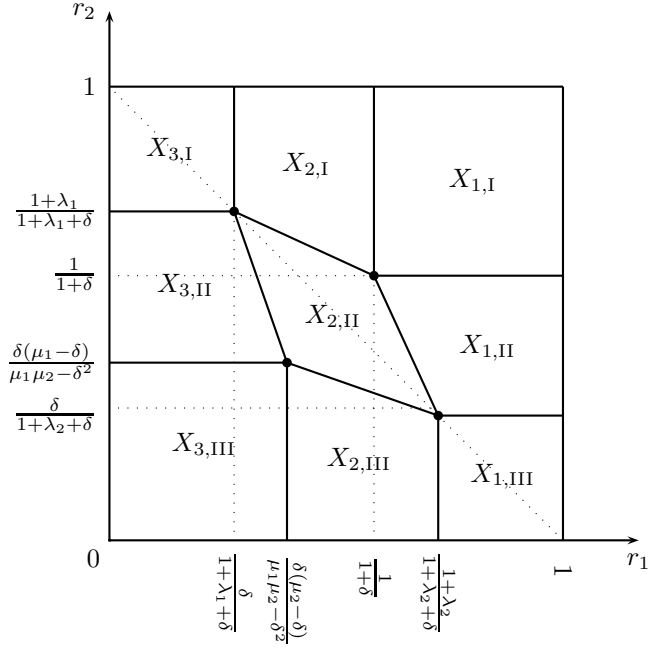


Figure 1: The partition $X_{1,I}, \dots, X_{3,III}$.

on player 2's reference point. In $X_{2,II}$ the proposals depend on both players' reference points. Thus, only intermediate valued reference points turn up in the equilibrium proposals explicitly. An equilibrium share is above the reference point if this is low, and below the reference point if this is high.

We now define strategies \hat{f} for player 1 and \hat{g} for player 2 based on the sets X_ω and associated proposals x^ω, y^ω , where $\omega \in \{1, I, \dots, 3, III\}$. Consider player 1. At any odd moment t and for any reference point (r_1, r_2) , take the (unique) X_ω containing (r_1, r_2) : then player 1 makes the corresponding proposal x^ω . At any even moment t and for any reference point (r_1, r_2) , take again the relevant set X_ω : then player 1 accepts a proposal z if and only if $z_1 \geq y_1^\omega$. This defines the strategy \hat{f} . The strategy \hat{g} for player 2 is defined similarly.

We now have the following result.

Theorem 3.1 *The strategy profile (\hat{f}, \hat{g}) is an SPE. The outcome is*

$$x^{3,III} = \left(\frac{\mu_1(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2}, \frac{\delta(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} \right).$$

The proof of Theorem 3.1 is in Appendix A.

4 A uniqueness result

Let (f, g) be a pair of strategies and consider the following possible conditions on (f, g) .

- (U1) f and g are *stationary Markov strategies*. Specifically, the proposal prescribed by f at each odd moment is time independent and depends only on the reference points at that moment, and the Y/N decision prescribed by f at each even moment depends only on player 2's proposal and the reference points at that moment. Similarly for g .
- (U2) *Immediate acceptance*. Any proposal made by player 1 according to f is accepted by player 2 according to g , and conversely.
- (U3) *Accept-reject indifference*. According to (f, g) , player 2 is indifferent between accepting (Y) or rejecting (N) a proposal made by player 1, and conversely.

Note that these conditions are satisfied by the subgame perfect equilibrium in the classical model (without loss aversion). The equilibrium (\hat{f}, \hat{g}) in Theorem 3.1 was in fact constructed using these assumptions, which makes the following result intuitive.

Theorem 4.1 (\hat{f}, \hat{g}) is the unique SPE strategy profile satisfying (U1), (U2), and (U3).

The formal (lengthy) proof is presented in Appendix B. We now proceed with a discussion of these conditions.

The condition that the equilibrium strategies are stationary Markov strategies implies that they depend on the history of play of the game only through the effect this play has on the players' reference points. Note that this does not imply that the players are restricted to stationary Markov strategies: the SPE (\hat{f}, \hat{g}) is resistant to deviations also with other strategies.

The second condition means that any proposal is immediately accepted. We can in fact show that any SPE must satisfy this condition in *some* subgames, namely those where the reference points are higher than the (equilibrium) shares of the pie. This is intuitive, cf. the extreme case where the reference points are equal to 1 and, thus, are fixed throughout the rest of the game, so that such a subgame is equivalent to the classical game without loss aversion.

The third condition, finally, requires that each player is indifferent between accepting and rejecting the proposal made by the other player. Note that it follows from the basic equilibrium condition that accepting a proposal must make a player at least as well off as rejecting it. In the classical model with loss neutral players a player j cannot be strictly better off since otherwise the proposing player i could lower the share of j in the proposal and, thus, increase his own share and be better off. This argument, however is based on the fact that in the classical model the payoffs of the players do not change as a result of playing the game. In our model we cannot exclude the possibility that such a deviation by the proposing player i may lead to rejection by j since j 's new reference point resulting from the rejection may be lower as it would have been from rejecting the equilibrium proposal. This may effect not only player j 's future payoff but also the play of the game after rejection.

Based on many failed attempts to construct different equilibria we are inclined to think that the conditions (U1)–(U3) are implied by SPE, but, as mentioned, the question is still open.

5 Analysis of the equilibrium

In this section we analyze the SPE (\hat{f}, \hat{g}) with respect to the loss aversion coefficients λ_1 and λ_2 and the probability of continuation δ . We consider what happens if δ goes to 1 and if the players have different δ 's. We also consider what happens if the time lapse between proposals tends to zero, and establish a relation with asymmetric Nash bargaining solutions.

5.1 Comparative statics of the loss aversion coefficients

The result of playing the equilibrium (\hat{f}, \hat{g}) is the proposal and immediate acceptance of some distribution of the pie. Here we investigate the dependence of this distribution on the players' loss aversion coefficients. We restrict ourselves to the set of reference point pairs $X_{3,\text{III}}$, since this is the relevant set at the beginning of the game. Moreover, the comparative statics results in subgames where the reference points are in different sets X_ω , are similar. Recall that

$$x^{3,\text{III}} = \left(\frac{\mu_1(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2}, \frac{\delta(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} \right)$$

where $\mu_i = 1 + \lambda_i(1 - \delta)$. It is sufficient to restrict the analysis to one player, because what one player gains is exactly what the other player loses. Differentiating with respect to λ_1 and λ_2 yields

$$\frac{dx_1^{3,\text{III}}}{d\lambda_1} = -\frac{\delta^2(1 - \delta)^2(1 + \lambda_2)}{(\mu_1\mu_2 - \delta^2)^2} < 0,$$

and

$$\frac{dx_1^{3,\text{III}}}{d\lambda_2} = \frac{\delta\mu_1(1 - \delta)^2(1 + \lambda_1)}{(\mu_1\mu_2 - \delta^2)^2} > 0.$$

Thus, players are hurt by their own loss aversion and benefit from their opponent's. This result also holds for the other subgames. However, if a player's reference point is very high, i.e., $r_1 \geq x_1^\omega$ where $\omega \in \{1,\text{I}, 1,\text{II}, 1,\text{III}\}$ or $r_2 \geq y_2^\omega$ where $\omega \in \{3,\text{I}, 2,\text{I}, 1,\text{I}\}$, then the equilibrium is independent of this player's degree of loss aversion. For instance, if $(r_1, r_2) \in X_{1,\text{II}}$, then λ_1 has no influence on the equilibrium partition, but λ_2 has a positive effect on player 1's payoff and a negative effect on player 2's.

5.2 Convergence results with respect to the probability of continuation

5.2.1 Convergence of the SPE for a common δ

The question we consider here is what happens to the equilibrium if δ tends to one. Using l'Hôpital's rule we derive

$$\lim_{\delta \rightarrow 1} x_1^{3,\text{III}} = \frac{1 + \lambda_2}{2 + \lambda_1 + \lambda_2},$$

and hence

$$\lim_{\delta \rightarrow 1} x_2^{3,\text{III}} = \frac{1 + \lambda_1}{2 + \lambda_1 + \lambda_2}.$$

Observe that player i 's payoff is conversely proportional to $1 + \lambda_j$, where $i, j \in \{1, 2\}$ and $i \neq j$.

We can repeat this for all subgames. Figure 2 shows the nine sets of Figure 1 in the limit for δ going to 1 and the limit equilibrium proposals, for the case where $\lambda_2 > \lambda_1$. The limit outcome in $X_{1,\text{I}}$ is an equal split of the pie,

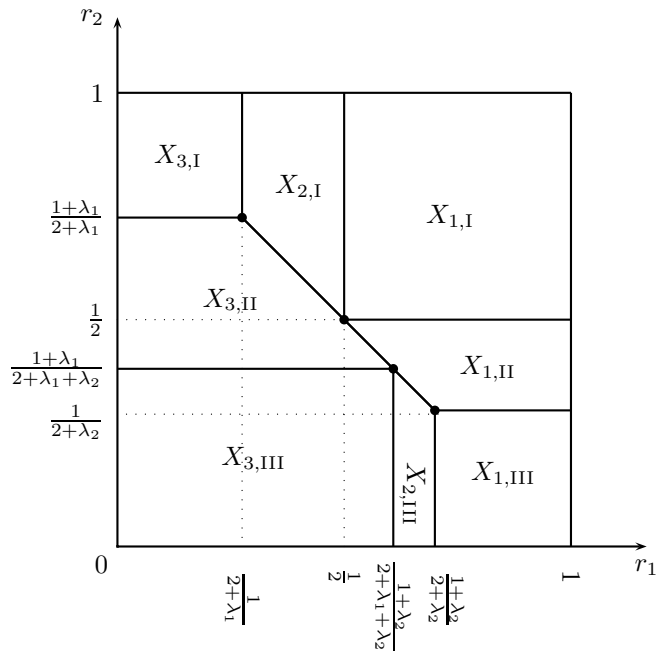


Figure 2: The equilibrium partitions for $\delta \rightarrow 1$, with $\lambda_2 > \lambda_1$.

$(1/2, 1/2)$. In $X_{3,\text{II}}$ and $X_{1,\text{II}}$ the limit equilibrium partition is $(1 - r_2, r_2)$, while in $X_{2,\text{III}}$ and $X_{2,\text{I}}$ it is $(r_1, 1 - r_1)$. In $X_{3,\text{I}}$ the limit equilibrium partition is $(1/(2 + \lambda_1), (1 + \lambda_1)/(2 + \lambda_1))$, and in $X_{1,\text{III}}$ it is $((1 + \lambda_2)/(2 + \lambda_2)$,

$1/(2 + \lambda_2)$). Set $X_{2,II}$ collapses to the line piece $r_1 + r_2 = 1$ where $r_1 \in (1/(2 + \lambda_1), (1 + \lambda_2)/(2 + \lambda_2))$. The limit equilibrium partition associated with this set is (r_1, r_2) .

5.2.2 Convergence of the SPE for $\delta_1 \neq \delta_2$

We generalize the model to the situation where the players have individual continuation probabilities δ_1 and δ_2 . In our setting, δ_i is interpreted as the probability that the game continues after player i rejected j 's proposal. Inequalities (2) and (3) generalize to

$$(1 + \lambda_2)x_2 - \lambda_2 \max\{r_2, x_2\} \geq \delta_2((1 + \lambda_2)y_2 - \lambda_2 \max\{y_2, \max\{r_2, x_2\}\}) - (1 - \delta_2)\lambda_2 \max\{r_2, x_2\}$$

and

$$(1 + \lambda_1)y_1 - \lambda_1 \max\{r_1, y_1\} \geq \delta_1((1 + \lambda_2)y_2 - \lambda_2 \max\{y_2, \max\{r_2, x_2\}\}) - (1 - \delta_1)\lambda_2 \max\{r_2, x_2\}.$$

All prior results also apply to this more general model. In particular, we obtain a solution to the above inequalities, considered as equalities, from which we construct a strategy profile. This strategy profile is the unique SPE satisfying (U1)–(U3).

A further generalization that leaves the previous results intact, is when there is a time lapse Δ between proposals, and the waiting time for breakdown of the game after player i , $i = 1, 2$, rejected the last proposal, is exponentially distributed with survival rate β_i . Then the probability that the game continues after player i rejected a proposal is δ_i^Δ , where $\delta_i = \exp(-1/\beta_i)$. Since the game starts with the reference points in $X_{3,III}$, we limit the analysis to this case. The SPE outcome is

$$x^{3,III} = \left(\frac{\mu_1(\mu_2 - \delta_2^\Delta)}{\mu_1\mu_2 - \delta_1^\Delta\delta_2^\Delta}, \frac{\delta_2^\Delta(\mu_1 - \delta_1^\Delta)}{\mu_1\mu_2 - \delta_1^\Delta\delta_2^\Delta} \right)$$

$$y^{3,III} = \left(\frac{\delta_1^\Delta(\mu_2 - \delta_2^\Delta)}{\mu_1\mu_2 - \delta_1^\Delta\delta_2^\Delta}, \frac{\mu_2(\mu_1 - \delta_1^\Delta)}{\mu_1\mu_2 - \delta_1^\Delta\delta_2^\Delta} \right)$$

where now $\mu_i := 1 + \lambda_i(1 - \delta_i^\Delta)$ for $i = 1, 2$. We can derive

$$\lim_{\Delta \rightarrow 0} x^{3,III} = \left(\frac{(1 + \lambda_2) \log \delta_2}{(1 + \lambda_1) \log \delta_1 + (1 + \lambda_2) \log \delta_2}, \frac{(1 + \lambda_1) \log \delta_1}{(1 + \lambda_1) \log \delta_1 + (1 + \lambda_2) \log \delta_2} \right).$$

In a similar way we obtain

$$\lim_{\Delta \rightarrow 0} y^{3,III} = \lim_{\Delta \rightarrow 0} x^{3,III}.$$

Note that this is an asymmetric Nash bargaining solution as described by Harsanyi and Selten (1972) and Kalai (1977). That is, it is the solution to the optimization problem

$$\arg \max_{z \in Z} z_1^\alpha z_2^{1-\alpha},$$

where

$$\alpha = \frac{(1 + \lambda_2) \log \delta_2}{(1 + \lambda_1) \log \delta_1 + (1 + \lambda_2) \log \delta_2}. \quad (4)$$

In Appendix C, we extend this result to the situation where players have concave utility functions $v_i : [0, 1] \rightarrow [0, 1]$, and the feasible set associated with the partitions of the pie is

$$\tilde{Z} := \{(v_1(\gamma), v_2(1 - \gamma)) \mid 0 \leq \gamma \leq 1\}.$$

We construct an SPE, resulting in an $x \in \tilde{Z}$, which is a function of δ_1^Δ and δ_2^Δ . Using an argument from Binmore *et al.* (1986) we show that if Δ goes to zero, then the SPE outcome x converges to \tilde{z}^N where

$$\tilde{z}^N := \arg \max_{z \in \tilde{Z}} z_1^\alpha z_2^{1-\alpha},$$

with α as defined by (4). Thus, increased loss aversion of a player results in increased ‘bargaining power’ of the opponent.

6 Concluding Remarks

In this paper we have investigated the effect of loss aversion in the strategic bargaining game of alternating offers introduced by Rubinstein (1982), by constructing a subgame perfect equilibrium in the extended model, and performing a comparative statics analysis on the outcome with respect to the bargainers’ loss aversion coefficients. We find that being loss averse has a negative effect on a player’s equilibrium share. We further find that the outcome of the bargaining procedure converges to an asymmetric Nash bargaining solution if the exogenous probability of breakdown goes to zero, such that higher loss aversion leads to higher bargaining power of the opponent.

This subgame perfect equilibrium is the unique SPE that shares three distinguishing features – stationary Markov strategies, immediate acceptance, and accept-reject indifference – with the unique SPE of Rubinstein. It is an open question whether uniqueness also holds without all or some of these conditions.

A The formal model and the SPE

In this section we formally complete the description of the model and show that the constructed strategy profile (\hat{f}, \hat{g}) is an SPE (i.e., we prove Theorem 3.1).

A.1 Formal model

We divide $T = \mathbb{N}$ into $T_{\text{odd}} := \{1, 3, \dots\}$ and $T_{\text{even}} := \{2, 4, \dots\}$. We assume that players have full information about the history of play: at any time $t \in T$, they know all previous proposals, their own as well as those of the other player.

Define h^t , the *history* of the game at time $t \in T$, as a vector of players' proposals which have taken place before and at time t . Specifically, $h^t := (z^1, \dots, z^t)$, where $z^s \in Z$ for all $s \leq t$. Furthermore, define H^t as the set of all possible histories h^t of the bargaining procedure at time $t \in T$. That is, $H^t := \prod_{s=1}^t Z$. Furthermore, let $H^0 := \{h^0\}$, where h^0 is the *empty history*. Henceforth, the term 'history' is used to indicate *non-empty* histories.

The players' *strategies* are elements in F or G where F is the set of infinite sequences of functions $(f^t)_{t \in T}$ where

$$\begin{aligned} \text{for } t = 1 : & \quad f^t \in Z, \\ \text{for } t > 1 \text{ and } t \in T_{\text{odd}} : & \quad f^t : H^{t-1} \rightarrow Z, \\ \text{for } t \in T_{\text{even}} : & \quad f^t : H^t \rightarrow \{Y, N\}, \end{aligned}$$

and G the set of infinite sequences of functions $(g^t)_{t \in T}$ where

$$\begin{aligned} \text{for } t \in T_{\text{odd}} : & \quad g^t : H^t \rightarrow \{Y, N\}, \\ \text{for } t \in T_{\text{even}} : & \quad g^t : H^{t-1} \rightarrow Z. \end{aligned}$$

An *agreement path* (h^t, a) is a history $h^t \in H^t$ ending in acceptance of the time t proposal. The set

$$A^t := \{(h^t, a) \mid h^t \in H^t\}$$

contains all time t agreement paths. The set

$$A := \bigcup_{t \in T} A^t$$

contains all histories ending in agreement. Similarly, we define a *disagreement path* (h^t, d) as a history $h^t \in H^t$ ending in breakdown of the negotiations upon rejection of the time t proposal. We define $D^t := \{(h^t, d) \mid h^t \in H^t\}$ and $D := \bigcup_{t \in T} D^t$. The set C^t contains all objects of the form (h^t, c) , i.e. histories that do not end at time t . Finally, we define

$$H^\infty := \{(z^1, z^2, \dots) \mid z^t \in Z \text{ for all } t \in T\},$$

and refer to elements of H^∞ as *infinite paths*. Then

$$\bar{H} := H^\infty \cup A \cup D$$

is the set containing all paths of the game. Observe that a strategy profile $(f, g) \in F \times G$ determines a specific play of the game or, equivalently, a set of paths in \bar{H} . In particular, if (f, g) leads to agreement on a partition at time t , then the set of paths associated with that strategy profile contains $t - 1$ paths in D and one in A . If (f, g) never induces an agreement, then this set contains a single path in H^∞ , and countably many in D .

We introduce the function $\xi_i : \bar{H} \setminus H^\infty \rightarrow [0, 1]$ that specifies for each finite path in \bar{H} the (physical) share of the pie bargainer i obtains. Specifically, for all $h^t \in H$, $h^t = (z^1, \dots, z^t)$, we have

$$\xi_i(h^t, a) := z_i^t, \text{ and}$$

$$\xi_i(h^t, d) := 0.$$

We define player i 's *utility function* for (dis)agreement paths as

$$u_i(h^t, a) := w(\xi_i(h^t, a), r_i(h^t), \lambda_i), \quad u_i(h^t, d) := w(\xi_i(h^t, d), r_i(h^t), \lambda_i).$$

Furthermore, player i 's utility evaluation of paths in H^∞ , i.e., perpetual disagreement, is defined as $-\lambda_i$. That is,

$$u_i(h) := -\lambda_i \text{ for all } h \in H^\infty.$$

Finally, we define the *expected* utility function $U_i : F \times G \rightarrow \mathbb{R}$. Let $t \in T$ be the point in time up until which the history is known, and let players play the strategy profile $(f, g) \in F \times G$ from then on. Then we say they play $(f|h^t, g|h^t)$ at time $t + 1$, and we denote by $U_i(f|h^t, g|h^t)$ player i 's time t expected utility from the strategy profile $(f, g) \in F \times G$. This can be exactly calculated, and is known to both players.

The strategy profile (f, g) is a *subgame perfect equilibrium* (SPE) if for every $t \in T$ and every $h^t \in H^t$, we have

$$U_1(f|h^t, g|h^t) \geq U_1(\tilde{f}|h^t, g|h^t) \text{ for all } \tilde{f}, \text{ and}$$

$$U_2(f|h^t, g|h^t) \geq U_2(f|h^t, \tilde{g}|h^t) \text{ for all } \tilde{g}.$$

A.2 Proof of Theorem 3.1

We first formally describe the nine sets, also referred to as regions, and associated proposals on which the definition of (\hat{f}, \hat{g}) is based.

A.2.1 Definition of X_ω and x^ω, y^ω

• Region 1,I

$$X_{1,I} = \left\{ (r_1, r_2) \in [0, 1]^2 \mid r_1 > \frac{1}{1+\delta}, r_2 > \frac{1}{1+\delta} \right\}.$$

The equilibrium proposals in $X_{1,I}$ are

$$x^{1,I} = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta} \right), \quad y^{1,I} = \left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta} \right).$$

[Thus, if both players' reference points lie above $1/(1+\delta)$, then the regular Rubinstein outcomes are obtained. In Figure 1, the Rubinstein outcome is the South-West corner point of this set.]

• **Region 1,III**

$$X_{1,III} = \left\{ (r_1, r_2) \in [0, 1]^2 \mid r_1 > \frac{1+\lambda_2}{1+\lambda_2+\delta}, r_2 \leq \frac{\delta}{1+\lambda_2+\delta} \right\}.$$

The equilibrium proposals in $X_{1,III}$ are

$$x^{1,III} = \left(\frac{1+\lambda_2}{1+\lambda_2+\delta}, \frac{\delta}{1+\lambda_2+\delta} \right)$$

$$y^{1,III} = \left(\frac{\delta(1+\lambda_2)}{1+\lambda_2+\delta}, \frac{1+\lambda_2(1-\delta)}{1+\lambda_2+\delta} \right).$$

[Observe that this set is a square, and that its North-West corner point lies on the line $r_1 + r_2 = 1$.]

• **Region 3,I**

$$X_{3,I} = \left\{ (r_1, r_2) \in [0, 1]^2 \mid r_1 \leq \frac{\delta}{1+\lambda_1+\delta}, r_2 > \frac{1+\lambda_1}{1+\lambda_1+\delta} \right\}.$$

This set is similar to $X_{1,III}$, but with the roles of the players reversed. The equilibrium proposals are

$$x^{3,I} = \left(\frac{1+\lambda_1(1-\delta)}{1+\lambda_1+\delta}, \frac{\delta(1+\lambda_1)}{1+\lambda_1+\delta} \right)$$

$$y^{3,I} = \left(\frac{\delta}{1+\lambda_1+\delta}, \frac{1+\lambda_1}{1+\lambda_1+\delta} \right).$$

[This set is again a square, and its South-East corner point lies on the line $r_1 + r_2 = 1$.]

• **Region 3,III**

$$X_{3,III} = \left\{ (r_1, r_2) \in [0, 1]^2 \mid r_1 \leq \frac{\delta(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2}, r_2 \leq \frac{\delta(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} \right\}.$$

The equilibrium proposals are

$$x^{3,III} = \left(\frac{\mu_1(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2}, \frac{\delta(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} \right), \quad y^{3,III} = \left(\frac{\delta(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2}, \frac{\mu_2(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} \right).$$

• **Region 1,II**

$$X_{1,II} = \left\{ (r_1, r_2) \in [0, 1]^2 \mid r_1 > \frac{(\mu_2 - \delta) + \delta\lambda_2(1 - r_2)}{(1 + \lambda_2) - \delta^2}, \frac{\delta}{1 + \lambda_2 + \delta} < r_2 \leq \frac{1}{1 + \delta} \right\}.$$

The equilibrium proposals are

$$\begin{aligned} x^{1,II} &= \left(\frac{(\mu_2 - \delta) + \delta\lambda_2(1 - r_2)}{(1 + \lambda_2) - \delta^2}, \frac{\delta(1 - \delta) + \delta\lambda_2 r_2}{(1 + \lambda_2) - \delta^2} \right) \\ y^{1,II} &= \left(\frac{\delta(\mu_2 - \delta) + \delta^2\lambda_2(1 - r_2)}{(1 + \lambda_2) - \delta^2}, \frac{(\mu_2 - \delta) + \delta^2\lambda_2 r_2}{(1 + \lambda_2) - \delta^2} \right). \end{aligned}$$

[Observe that these proposals now depend on player 2's reference point.]

• **Region 3,II**

$$\begin{aligned} X_{3,II} &= \left\{ (r_1, r_2) \in [0, 1]^2 \mid r_1 \leq \frac{\delta(\mu_2 - \delta + \delta\lambda_2(1 - r_2))}{\mu_1(1 + \lambda_2) - \delta^2}, \right. \\ &\quad \left. \frac{\delta(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} < r_2 \leq \frac{1 + \lambda_1}{1 + \lambda_1 + \delta} \right\}. \end{aligned}$$

The equilibrium proposals are

$$\begin{aligned} x^{3,II} &= \left(\frac{\mu_1(\mu_2 - \delta + \lambda_2\delta(1 - r_2))}{\mu_1(1 + \lambda_2) - \delta^2}, \frac{\delta(\mu_1 - \delta + \lambda_2 r_2 \mu_1)}{\mu_1(1 + \lambda_2) - \delta^2} \right) \\ y^{3,II} &= \left(\frac{\delta(\mu_2 - \delta + \delta\lambda_2(1 - r_2))}{\mu_1(1 + \lambda_2) - \delta^2}, \frac{(\mu_1 - \delta)(1 + \lambda_2) + \delta^2\lambda_2 r_2}{\mu_1(1 + \lambda_2) - \delta^2} \right). \end{aligned}$$

[In this set player 1's reference point is low, which explains that the equilibrium proposals do not depend on it.]

• **Region 2,I**

$$X_{2,I} = \left\{ (r_1, r_2) \in [0, 1]^2 \mid \frac{\delta}{1 + \lambda_1 + \delta} < r_1 \leq \frac{1}{1 + \delta}, r_2 > \frac{(\mu_1 - \delta) + \delta\lambda_1(1 - r_1)}{(1 + \lambda_1) - \delta^2} \right\}.$$

The equilibrium proposals are

$$\begin{aligned} x^{2,I} &= \left(\frac{(\mu_1 - \delta) + \delta^2\lambda_1 r_1}{(1 + \lambda_1) - \delta^2}, \frac{\delta(\mu_1 - \delta) + \delta^2\lambda_1(1 - r_1)}{(1 + \lambda_1) - \delta^2} \right) \\ y^{2,I} &= \left(\frac{\delta(1 - \delta) + \delta\lambda_1 r_1}{(1 + \lambda_1) - \delta^2}, \frac{(\mu_1 - \delta) + \delta\lambda_1(1 - r_1)}{(1 + \lambda_1) - \delta^2} \right). \end{aligned}$$

[This set is similar to $X_{1,II}$.]

• **Region 2,III**

$$X_{2,III} = \left\{ (r_1, r_2) \in [0, 1]^2 \mid \frac{\delta(\mu_2 - \delta)}{\mu_1\mu_2 - \delta^2} \leq r_1 \leq \frac{1 + \lambda_2}{1 + \lambda_2 + \delta}, \right.$$

$$r_2 \leq \left. \frac{\delta((\mu_1 - \delta) + \delta\lambda_1(1 - r_1))}{(1 + \lambda_1)\mu_2 - \delta^2} \right\}.$$

The equilibrium proposals are

$$\begin{aligned} x^{2,\text{III}} &= \left(\frac{(1 + \lambda_1)(\mu_2 - \delta) + \delta^2\lambda_1 r_1}{(1 + \lambda_1)\mu_2 - \delta^2}, \frac{\delta((\mu_1 - \delta) + \delta\lambda_1(1 - r_1))}{(1 + \lambda_1)\mu_2 - \delta^2} \right) \\ y^{2,\text{III}} &= \left(\frac{\delta((\mu_2 - \delta) + \mu_2\lambda_1 r_1)}{(1 + \lambda_1)\mu_2 - \delta^2}, \frac{\mu_2((\mu_1 - \delta) + \delta\lambda_1(1 - r_1))}{(1 + \lambda_1)\mu_2 - \delta^2} \right). \end{aligned}$$

[In this set player 2's reference point is low, which explains that the equilibrium proposals do not depend on it.]

• **Region 2,II**

The boundaries of $X_{2,\text{II}}$ are described by the boundaries of the neighboring sets, see Figure 1. The equilibrium proposals $x^{2,\text{II}}$ and $y^{2,\text{II}}$ are given by

$$\begin{aligned} x_1^{2,\text{II}} &= \frac{(1 + \lambda_1)((\mu_2 - \delta) + \delta\lambda_2(1 - r_2)) + \delta^2\lambda_1 r_1}{(1 + \lambda_1)(1 + \lambda_2) - \delta^2}, \\ x_2^{2,\text{II}} &= \frac{\delta(\mu_1 - \delta + \delta\lambda_1(1 - r_1) + \lambda_2(1 + \lambda_1)r_2)}{(1 + \lambda_1)(1 + \lambda_2) - \delta^2}, \\ y_1^{2,\text{II}} &= \frac{\delta(\mu_2 - \delta + \delta\lambda_2(1 - r_2) + \lambda_1(1 + \lambda_2)r_1)}{(1 + \lambda_1)(1 + \lambda_2) - \delta^2}, \\ y_2^{2,\text{II}} &= \frac{(1 + \lambda_2)((\mu_1 - \delta) + \delta\lambda_1(1 - r_1)) + \delta^2\lambda_2 r_2}{(1 + \lambda_1)(1 + \lambda_2) - \delta^2}. \end{aligned}$$

A.2.2 Proof of Theorem 3.1

To prove Theorem 3.1, we use the *one-deviation property*. The one-deviation property says that for a strategy profile $(f, g) \in F \times G$ to be a subgame perfect equilibrium it is sufficient that no player can improve by deviating only once, i.e., at one point in time.

Hendon *et al.* (1996) showed that the one-deviation property holds in infinite-horizon extensive-form games which are *continuous at infinity*³. Continuity at infinity is defined as follows. For any $\varepsilon > 0$ there is a number $t \in T$ such that if two strategy profiles $(f, g), (f', g') \in F \times G$ are such that $(f^s, g^s) = (f'^s, g'^s)$ for all $s \leq t$, then $|U_i(f, g) - U_i(f', g')| < \varepsilon$.

Lemma A.1 *The bargaining game with loss averse players is continuous at infinity.*

Proof. Let $\varepsilon > 0$, and let (f, g) and (f', g') be strategy profiles in $F \times G$ satisfying $(f^s, g^s) = (f'^s, g'^s)$ for all $s \leq t$, where $t > \max_{i=1,2} \log_\delta \varepsilon / (1 + \lambda_i)$. Observe that the largest payoff difference between two such strategy profiles

³Another requirement concerns Bayesian updating, but this is automatically fulfilled since our game is one of perfect information.

would emerge when the one yields player i the whole pie at time $t + 1$, while the other leads to perpetual disagreement. In the former case, player i would obtain

$$\bar{U}_i = \delta^t + (1 - \delta) \sum_{s=1}^t \delta^{s-1} u_i(h^s, d),$$

while in the latter he would obtain

$$\underline{U}_i = (1 - \delta) \sum_{s=1}^{\infty} \delta^{s-1} u_i(h^s, d).$$

From the fact that $u_i(h^t, d) \geq -\lambda_i$ for all $t \in T$, it follows that

$$\begin{aligned} \underline{U}_i &= (1 - \delta) \sum_{s=1}^t \delta^{s-1} u_i(h^s, d) + (1 - \delta) \sum_{s=t+1}^{\infty} \delta^{s-1} u_i(h^s, d) \\ &\geq (1 - \delta) \sum_{s=1}^t \delta^{s-1} u_i(h^s, d) + \delta^t (1 - \delta) (-\lambda_i) \frac{1}{1 - \delta} \\ &= -\delta^t \lambda_i + (1 - \delta) \sum_{s=1}^t \delta^{s-1} u_i(h^s, d). \end{aligned}$$

From this and the fact that $\bar{U}_i - \underline{U}_i \geq 0$, we obtain

$$\begin{aligned} |U_i(f, g) - U_i(f', g')| &\leq \bar{U}_i - \underline{U}_i \leq \delta^t + (1 - \delta) \sum_{s=1}^t \delta^{s-1} u_i(h^s, d) \\ &\quad + \delta^t \lambda_i - (1 - \delta) \sum_{s=1}^t \delta^{s-1} u_i(h^s, d) = \delta^t (1 + \lambda_i) < \varepsilon. \end{aligned}$$

Hence, the game is continuous at infinity. \square

It follows from Lemma A.1 that we can use the one-deviation property.

Denote $\Omega = \{1.I., \dots, 3.III.\}$.

Proof of Theorem 3.1. To show that (\hat{f}, \hat{g}) is SPE, it is sufficient to show that there is no subgame in which player 1 (2) can profitably deviate from \hat{f} (\hat{g}) at a single time $t \in T$, given that player 2 (1) plays strategy \hat{g} (\hat{f}). Assume player 2 plays strategy \hat{g} . We denote the utility player 1 obtains by following strategy \hat{f} by u_1^* .

Let $h^{t-1} \in C^{t-1}$, i.e. h^{t-1} is a history continuing to the next period. Assume h^{t-1} is such that $(r_1(h^{t-1}), r_2(h^{t-1})) \in X_\omega$ with $\omega \in \Omega$, and that $h^t = (h^{t-1}, z)$ with $z \in Z$. If t is odd (even), then z is proposed by player 1 (2). If the proposal z is rejected, then the game continues with probability δ to the next period or ends with probability $1 - \delta$. If the game continues to time $t + 1$, then it ends

in immediate acceptance of the proposal at $t + 1$, since (\hat{f}, \hat{g}) is the prevalent strategy profile.

To show that \hat{f} is a best reply to \hat{g} we distinguish two cases, namely t is odd and t is even. For each case, we consider three subcases. The reference point pair (r_1, r_2) is in X_ω where

Case 1. $\omega \in \{1, \text{I}, 1, \text{II}, 1, \text{III}\}$. Then $r_1 > x_1^\omega > y_1^\omega$, and $y_1^\omega = \delta x_1^\omega$.

Case 2. $\omega \in \{2, \text{I}, 2, \text{II}, 2, \text{III}\}$. Then $x_1^\omega \geq r_1 > y_1^\omega$, and $y_1^\omega = \frac{\delta(x_1^\omega + \lambda_1 r_1)}{1 + \lambda_1}$.

Case 3. $\omega \in \{3, \text{I}, 3, \text{II}, 3, \text{III}\}$. Then $x_1^\omega > y_1^\omega \geq r_1$, and $y_1^\omega = \frac{\delta}{\mu_1} x_1^\omega$.

t odd, case 1. We distinguish between the following cases:

- $z_1 = x_1^\omega$: In this case player 1 follows strategy \hat{f} . Player 2 accepts, so

$$u_1^* = u_1(h^t, a) = (1 + \lambda_1)x_1^\omega - \lambda_1 \max\{r_1, x_1^\omega\} = (1 + \lambda_1)x_1^\omega - \lambda_1 r_1.$$

- $z_1 < x_1^\omega$: Then $z_2 > x_2^\omega$, so player 2 accepts. Player 1's payoff is $u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\}$. Observe that $r_1 \geq x_1^\omega > z_1$ implies that

$$u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 r_1.$$

Then $z_1 < x_1^\omega$ implies $u_1(h^t, a) < u_1^*$. Hence, the proposal z is not optimal.

- $z_1 > x_1^\omega$: Then $z_2 < x_2^\omega$, so player 2 rejects and proposes y^ω if the game continues. Hence, player 1 obtains

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= \delta((1 + \lambda_1)y_1^\omega - \lambda_1 \max\{y_1^\omega, r_1\}) - (1 - \delta)\lambda_1 r_1 \\ &= (1 + \lambda_1)\delta y_1^\omega - \delta\lambda_1 r_1 - (1 - \delta)\lambda_1 r_1 \\ &= (1 + \lambda_1)\delta y_1^\omega - \lambda_1 r_1. \end{aligned}$$

Since $x_1^\omega > \delta^2 x_1^\omega = \delta y_1^\omega$, we have $\delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) < u_1^*$. Thus, the proposal z is not optimal.

t odd, case 2. We distinguish between three cases:

- $z_1 = x_1^\omega$: In this case player 1 follows strategy \hat{f} . Player 2 accepts, so

$$u_1^* = u_1(h^t, a) = (1 + \lambda_1)x_1^\omega - \lambda_1 \max\{x_1^\omega, r_1\} = x_1^\omega.$$

- $z_1 < x_1^\omega$: Since then $z_2 > x_2^\omega$, player 2 accepts. Player 1's payoff is $u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\}$. From

$$(1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\} \leq z_1 < x_1^\omega = u_1^*,$$

it follows that the proposal z is not optimal.

- $z_1 > x_1^\omega$: Since then $z_2 < x_2^\omega$, player 2 rejects, and proposes y^ω if the game continues. Hence, player 1 obtains

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta y_1^\omega - \delta \lambda_1 \max\{y_1^\omega, r_1\} - (1 - \delta)\lambda_1 r_1 \\ &= (1 + \lambda_1)\delta y_1^\omega - \lambda_1 r_1. \end{aligned}$$

From $y_1^\omega = \frac{\delta(x_1^\omega + \lambda_1 r_1)}{1 + \lambda_1}$ it follows that $\delta x_1^\omega = (1 + \lambda_1)y_1^\omega - \delta \lambda_1 r_1$. Therefore,

$$u_1^* = x_1^\omega = (1 + \lambda_1)\frac{1}{\delta}y_1^\omega - \lambda_1 r_1 > (1 + \lambda_1)\delta y_1^\omega - \lambda_1 r_1.$$

Thus, the proposal z is not optimal.

t **odd, case 3.** We distinguish between three cases:

- $z_1 = x_1^\omega$: In this case player 1 follows strategy \hat{f} . Player 2 accepts, so

$$u_1^* = u_1(h^t, a) = (1 + \lambda_1)x_1^\omega - \lambda_1 \max\{x_1^\omega, r_1\} = x_1^\omega.$$

- $z_1 < x_1^\omega$: Since then $z_2 > x_2^\omega$, player 2 accepts. Player 1's payoff is $u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\}$. From

$$(1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\} \leq z_1 < x_1^\omega,$$

it follows that $u_1^* > u_1(h^t, a)$. Thus, the proposal z is not optimal.

- $z_1 > x_1^\omega$: Since $z_2 < x_2^\omega$, player 2 rejects, and proposes y^ω if the game continues. Hence, player 1 obtains

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta y_1^\omega - \delta \lambda_1 \max\{y_1^\omega, r_1\} - (1 - \delta)\lambda_1 r_1 \\ &= \delta y_1^\omega - (1 - \delta)\lambda_1 r_1. \end{aligned}$$

Observe that

$$u_1^* = x_1^\omega > y_1^\omega > \delta y_1^\omega \geq \delta y_1^\omega - (1 - \delta)\lambda_1 r_1.$$

Hence, the proposal z is not optimal.

t **even, case 1.** Assume player 2 proposes some $z \in Z$. Accepting gives player 1

$$u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\}.$$

Rejecting makes his reference point switch to $\max\{r_1, z_1\}$; if the game continues, he proposes x^ω , and player 2 accepts. Since $r_1 > x_1^\omega$ and $y_1^\omega = \delta x_1^\omega$, we have

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta x_1^\omega - \delta \lambda_1 \max\{x_1^\omega, \max\{r_1, z_1\}\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= (1 + \lambda_1)\delta x_1^\omega - \delta \lambda_1 \max\{r_1, z_1\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= (1 + \lambda_1)y_1^\omega - \lambda_1 \max\{r_1, z_1\}. \end{aligned}$$

Hence, it is optimal to accept the proposal z if $z_1 \geq y_1^\omega$, and to reject it otherwise. That is, to play strategy \hat{f} .

***t* even, case 2.** Assume player 2 proposes some $z \in Z$ with $z_1 \leq x_1^{\omega'}$ where $\omega' \in \{1, \text{I}, 1, \text{II}, 1, \text{III}\}$. Accepting gives player 1

$$u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\}.$$

If he rejects and the game continues, then the reference point $r_1(h^t)$ is $\max\{r_1, z_1\}$. Thus, at $t + 1$ we enter a new game in which $(\max\{r_1, z_1\}, r_2)$ is the prevalent reference point pair. Since $(\max\{r_1, z_1\}, r_2) \in \omega$ where $\omega \in \{2, \text{I}, 2, \text{II}, 2, \text{III}\}$, we have $x_1^\omega \geq \max\{r_1, z_1\} \geq y_1^\omega$, where $y_1^\omega = \delta(x_1^\omega + \lambda_1 \max\{r_1, z_1\}) / (1 + \lambda_1)$. At $t + 1$ player 1 proposes x_1^ω and player 2 accepts, so rejecting z yields

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta x_1^\omega - \delta \lambda_1 \max\{x_1^\omega, \max\{r_1, z_1\}\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= \delta x_1^\omega - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= (1 + \lambda_1)y_1^\omega - \lambda_1 \max\{r_1, z_1\}. \end{aligned}$$

If player 2 proposes $z \in Z$ with $z_1 > x_1^{\omega'}$, then accepting yields $u_1(h^t, a) = z_1$ and rejecting

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta x_1^{\omega'} - \delta \lambda_1 \max\{x_1^{\omega'}, \max\{r_1, z_1\}\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= (1 + \lambda_1)y_1^{\omega'} - \lambda_1 z_1. \end{aligned}$$

Since $z_1 > x_1^{\omega'} > y_1^{\omega'}$, we have $u_1(h^t, a) > \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d)$. In general, it is optimal to accept the proposal z if $z_1 \geq y_1^\omega$, and to reject it otherwise. That is, it is optimal to follow strategy \hat{f} .

***t* even, case 3.** Assume player 2 proposes some $z \in Z$ with $z_1 < y_1^\omega$. Note that $r_1 < x_1^\omega$. Hence, rejecting yields

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta x_1^\omega - \delta \lambda_1 \max\{x_1^\omega, \max\{r_1, z_1\}\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \quad (5) \\ &= \delta x_1^\omega - (1 - \delta)\lambda_1 \max\{r_1, z_1\}. \end{aligned}$$

Accepting yields $u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\}$. Since $y_1^\omega = \frac{\delta}{\mu_1}x_1^\omega$, we have

$$\begin{aligned} u_1(h^t, a) &= (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\} \\ &= (1 + \lambda_1)z_1 - \delta \lambda_1 \max\{r_1, z_1\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &< (1 + \lambda_1)y_1^\omega - \delta \lambda_1 \max\{r_1, y_1^\omega\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \quad (6) \\ &= (1 + \lambda_1)y_1^\omega - \delta \lambda_1 y_1^\omega - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= \mu_1 y_1^\omega - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= \delta x_1^\omega - (1 - \delta)\lambda_1 \max\{r_1, z_1\}. \end{aligned}$$

Thus, it is optimal to reject z . If player 2 proposes $z \in Z$ with $z_1 \geq y_1^\omega$, then accepting yields

$$u_1(h^t, a) = (1 + \lambda_1)z_1 - \lambda_1 \max\{r_1, z_1\} = z_1.$$

Let $z_1 \leq y_1^{\omega'}$ where $\omega' \in \{1, \text{I}, 1, \text{II}, 1, \text{III}\}$. If player 1 rejects and the game continues, then $r_1(h^t) = z_1$, and $(z_1, r_2) \in \omega''$ where $\omega'' \in \{2, \text{I}, 2, \text{II}, 2, \text{III}\}$. Note that then $y_1^{\omega''} \leq z_1 \leq x_1^{\omega''}$. Thus, rejecting yields

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta x_1^{\omega''} - \delta \lambda_1 \max\{x_1^{\omega''}, z_1\} - (1 - \delta)\lambda_1 z_1 \\ &= \delta x_1^{\omega''} - (1 - \delta)\lambda_1 z_1 \\ &= (1 + \lambda_1)y_1^{\omega''} - \lambda_1 z_1. \end{aligned} \tag{7}$$

Since $z_1 \geq y_1^{\omega''}$, also $z_1 \geq (1 + \lambda_1)y_1^{\omega''} - \lambda_1 z_1$. If player 2 proposes $z \in Z$ with $z_1 > x_1^{\omega'}$, then rejecting yields

$$\begin{aligned} & \delta u_1(h^{t+1}, a) + (1 - \delta)u_1(h^t, d) \\ &= (1 + \lambda_1)\delta x_1^{\omega'} - \delta \lambda_1 \max\{x_1^{\omega'}, \max\{r_1, z_1\}\} - (1 - \delta)\lambda_1 \max\{r_1, z_1\} \\ &= (1 + \lambda_1)\delta x_1^{\omega'} - \delta \lambda_1 z_1 - (1 - \delta)\lambda_1 z_1 \\ &= (1 + \lambda_1)\delta x_1^{\omega'} - \lambda_1 z_1. \end{aligned} \tag{8}$$

Observe that $z_1 > x_1^{\omega'}$ implies $z_1 > \delta x_1^{\omega'}$. It follows that $z_1 > (1 + \lambda_1)\delta x_1^{\omega'} - \lambda_1 z_1$, i.e. accepting z is optimal.

Thus, we have shown that player 1 can not profitably deviate from \hat{f} at any single time t , given that player 2 plays strategy \hat{g} . The proof that player 2 can not profitably deviate from \hat{g} at any single time t , given that player 1 plays \hat{f} , is analogous. Lemma A.1 implies that (\hat{f}, \hat{g}) is an SPE. \square

B Proof of Theorem 4.1

Throughout this section, we assume that conditions (U1)–(U3) are satisfied.

B.1 Preliminary lemmas

Define a bargaining round as an offer made by player 1 and a counter offer made by player 2. Bargaining rounds are indexed with $i \in \{0, 1, 2, \dots\}$. Then (r_1^i, r_2^i) is the reference point pair prevalent at the beginning of bargaining round i . In view of (U1) it makes sense to define

$$r_1^{i+1} := \max\{r_1^i, y_1(r_1^i, r_2^{i+1})\}, \text{ and } r_2^{i+1} := \max\{r_2^i, x_2(r_1^i, r_2^i)\}. \quad (9)$$

This allows us to show the following.

Lemma B.1 *When $x(r_1^i, r_2^i)$ is player 1's SPE proposal and $y(r_1^i, r_2^{i+1})$ player 2's counterproposal, we have*

$$x_2(r_1^i, r_2^i) = \begin{cases} \delta - \delta y_1(r_1^i, r_2^{i+1}) & \text{if } r_2^i > y_2(r_1^i, r_2^{i+1}) > x_2(r_1^i, r_2^i) \\ \frac{\delta(1+\lambda_2 r_2^{i+1})}{1+\lambda_2} - \frac{\delta y_1(r_1^i, r_2^{i+1})}{1+\lambda_2} & \text{if } y_2(r_1^i, r_2^{i+1}) \geq r_2^i > x_2(r_1^i, r_2^i) \\ \frac{\delta}{\mu_2} - \frac{\delta}{\mu_2} y_1(r_1^i, r_2^{i+1}) & \text{if } y_2(r_1^i, r_2^{i+1}) > x_2(r_1^i, r_2^i) \geq r_2^i, \end{cases}$$

and

$$y_1(r_1^i, r_2^{i+1}) = \begin{cases} \delta - \delta x_2(r_1^{i+1}, r_2^{i+1}) & \text{if } r_1^i > x_1(r_1^{i+1}, r_2^{i+1}) > y_1(r_1^i, r_2^{i+1}) \\ \frac{\delta(1+\lambda_1 r_1^{i+1})}{1+\lambda_1} - \frac{\delta x_2(r_1^{i+1}, r_2^{i+1})}{1+\lambda_1} & \text{if } x_1(r_1^{i+1}, r_2^{i+1}) \geq r_1^i > y_1(r_1^i, r_2^{i+1}) \\ \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2(r_1^{i+1}, r_2^{i+1}) & \text{if } x_1(r_1^{i+1}, r_2^{i+1}) > y_1(r_1^i, r_2^{i+1}) \geq r_1^i. \end{cases}$$

Proof. This follows from (U2) and (U3), and the definition of the players' utility functions. \square

For each $\omega \in \Omega$ we introduce the following sets P_ω and Q_ω of pairs of reference points:

$$P_{1,I} := \{(r_1^i, r_2^i) \mid r_2^i > y_2(r_1^i, r_2^{i+1}), r_1^i > x_1(r_1^{i+1}, r_2^{i+1})\}$$

$$P_{2,I} := \{(r_1^i, r_2^i) \mid r_2^i > y_2(r_1^i, r_2^{i+1}), x_1(r_1^{i+1}, r_2^{i+1}) \geq r_1^i > y_1(r_1^i, r_2^{i+1})\}$$

$$P_{3,I} := \{(r_1^i, r_2^i) \mid r_2^i > y_2(r_1^i, r_2^{i+1}), y_1(r_1^i, r_2^{i+1}) \geq r_1^i\}$$

$$P_{1,II} := \{(r_1^i, r_2^i) \mid y_2(r_1^i, r_2^{i+1}) \geq r_2^i > x_2(r_1^i, r_2^i), r_1^i > x_1(r_1^{i+1}, r_2^{i+1})\}$$

$$P_{2,II} := \{(r_1^i, r_2^i) \mid y_2(r_1^i, r_2^{i+1}) \geq r_2^i > x_2(r_1^i, r_2^i), x_1(r_1^{i+1}, r_2^{i+1}) \geq r_1^i > y_1(r_1^i, r_2^{i+1})\}$$

$$P_{3,\text{II}} := \{(r_1^i, r_2^i) \mid y_2(r_1^i, r_2^{i+1}) \geq r_2^i > x_2(r_1^i, r_2^i), y_1(r_1^i, r_2^{i+1}) \geq r_1^i\}$$

$$P_{1,\text{III}} := \{(r_1^i, r_2^i) \mid x_2(r_1^i, r_2^i) \geq r_2^i, r_1^i > x_1(r_1^{i+1}, r_2^{i+1})\}$$

$$P_{2,\text{III}} := \{(r_1^i, r_2^i) \mid x_2(r_1^i, r_2^i) \geq r_2^i, x_1(r_1^{i+1}, r_2^{i+1}) \geq r_1^i > y_1(r_1^i, r_2^{i+1})\}$$

$$P_{3,\text{III}} := \{(r_1^i, r_2^i) \mid x_2(r_1^i, r_2^i) \geq r_2^i, y_1(r_1^i, r_2^{i+1}) \geq r_1^i\}$$

$$Q_{1,\text{I}} := \{(r_1^i, r_2^{i+1}) \mid r_1^i > x_1(r_1^{i+1}, r_2^{i+1}), r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2})\}$$

$$Q_{1,\text{II}} := \{(r_1^i, r_2^{i+1}) \mid r_1^i > x_1(r_1^{i+1}, r_2^{i+1}), y_2(r_1^{i+1}, r_2^{i+2}) \geq r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})\}$$

$$Q_{1,\text{III}} := \{(r_1^i, r_2^{i+1}) \mid r_1^i > x_1(r_1^{i+1}, r_2^{i+1}), x_2(r_1^{i+1}, r_2^{i+1}) \geq r_2^{i+1}\}$$

$$Q_{2,\text{I}} := \{(r_1^i, r_2^{i+1}) \mid x_1(r_1^{i+1}, r_2^{i+1}) \geq r_1^i > y_1(r_1^i, r_2^{i+1}), r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2})\}$$

$$Q_{2,\text{II}} := \{(r_1^i, r_2^{i+1}) \mid x_1(r_1^{i+1}, r_2^{i+1}) \geq r_1^i > y_1(r_1^i, r_2^{i+1}), y_2(r_1^{i+1}, r_2^{i+2}) \geq r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})\}$$

$$Q_{2,\text{III}} := \{(r_1^i, r_2^{i+1}) \mid x_1(r_1^{i+1}, r_2^{i+1}) \geq r_1^i > y_1(r_1^i, r_2^{i+1}), x_2(r_1^{i+1}, r_2^{i+1}) \geq r_2^{i+1}\}$$

$$Q_{3,\text{I}} := \{(r_1^i, r_2^{i+1}) \mid y_1(r_1^i, r_2^{i+1}) \geq r_1^i, r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2})\}$$

$$Q_{3,\text{II}} := \{(r_1^i, r_2^{i+1}) \mid y_1(r_1^i, r_2^{i+1}) \geq r_1^i, y_2(r_1^{i+1}, r_2^{i+2}) \geq r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})\}$$

$$Q_{3,\text{III}} := \{(r_1^i, r_2^{i+1}) \mid y_1(r_1^i, r_2^{i+1}) \geq r_1^i, x_2(r_1^{i+1}, r_2^{i+1}) \geq r_2^{i+1}\}.$$

We derive a series of lemmas for reference point pairs in these sets.

Lemma B.2 *For all $(r_1^i, r_2^i) \in P_\omega$, we have*

$$x_2(r_1^i, r_2^i) \geq x_2(r_1^{i+1}, r_2^{i+1}) \Leftrightarrow x_2(r_1^i, r_2^i) \leq x_2^\omega.$$

Similarly, for all $(r_1^i, r_2^{i+1}) \in Q_\omega$, we have

$$y_1(r_1^i, r_2^{i+1}) \geq y_1(r_1^{i+1}, r_2^{i+2}) \Leftrightarrow y_1(r_1^i, r_2^{i+1}) \leq y_1^\omega.$$

Proof. Let $\omega = 1, \text{I}$ and $(r_1^i, r_2^i) \in P_\omega$. By definition of $P_{1,\text{I}}$ we have

$$r_2^i > y_2(r_1^i, r_2^{i+1}) \text{ and } r_1^i > x_1(r_1^{i+1}, r_2^{i+1}).$$

From Lemma B.1 we obtain

$$\begin{aligned} x_2(r_1^i, r_2^i) &= \delta - \delta y_1(r_1^i, r_2^{i+1}) \\ &= \delta - \delta(\delta - \delta x_2(r_1^{i+1}, r_2^{i+1})) \\ &= \delta - \delta^2 + \delta^2 x_2(r_1^{i+1}, r_2^{i+1}). \end{aligned} \tag{10}$$

Suppose $x_2(r_1^{i+1}, r_2^{i+1}) \geq x_2(r_1^i, r_2^i)$. Then by (10)

$$x_2(r_1^i, r_2^i) = \delta - \delta^2 + \delta^2 x_2(r_1^{i+1}, r_2^{i+1}) \geq \delta - \delta^2 + \delta^2 x_2(r_1^i, r_2^i).$$

It follows that

$$x_2(r_1^i, r_2^i) \geq \frac{\delta - \delta^2}{1 - \delta^2} = \frac{\delta}{1 + \delta} = x_2^{1,I}.$$

Suppose $x_2(r_1^{i+1}, r_2^{i+1}) < x_2(r_1^i, r_2^i)$. Then by (10)

$$x_2(r_1^i, r_2^i) = \delta - \delta^2 + \delta^2 x_2(r_1^{i+1}, r_2^{i+1}) < \delta - \delta^2 + \delta^2 x_2(r_1^i, r_2^i),$$

implying $x_2(r_1^i, r_2^i) < x_2^{1,I}$. Therefore,

$$x_2(r_1^i, r_2^i) \geq x_2(r_1^{i+1}, r_2^{i+1}) \Leftrightarrow x_2(r_1^i, r_2^i) \leq x_2^{1,I}.$$

Similarly, let $\omega = 1, I$ and $(r_1^i, r_2^{i+1}) \in Q_\omega$. By definition of $Q_{1,I}$

$$r_1^i > x_1(r_1^{i+1}, r_2^{i+1}) \text{ and } r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2}).$$

Then by Lemma B.1

$$\begin{aligned} y_1(r_1^i, r_2^{i+1}) &= \delta - \delta x_2(r_1^{i+1}, r_2^{i+1}) \\ &= \delta - \delta(\delta - \delta y_1(r_1^{i+1}, r_2^{i+2})) \\ &= \delta - \delta^2 + \delta^2 y_1(r_1^{i+1}, r_2^{i+2}), \end{aligned}$$

which implies

$$y_1(r_1^i, r_2^{i+1}) \geq y_1(r_1^{i+1}, r_2^{i+2}) \Leftrightarrow y_1(r_1^i, r_2^{i+1}) \leq y_1^{1,I}.$$

The proof for $\omega \in \Omega$, $\omega \neq 1, I$, is analogous. \square

The following lemma says that if the reference point pair is in P_ω now and at all future odd time points, then player 1 must propose x^ω . A similar result holds for Q_ω .

Lemma B.3 *If $(r_1^k, r_2^k) \in P_\omega$ for all $k \geq i$, then $x_2(r_1^i, r_2^i) = x_2^\omega$. Similarly, if $(r_1^k, r_2^{k+1}) \in Q_\omega$ for all $k \geq i$, then $y_1(r_1^i, r_2^{i+1}) = y_1^\omega$.*

Proof. Observe that if $(r_1^k, r_2^k) \in P_\omega$ for all $k \geq i$, then by Lemma B.1 it follows that $x_2(r_1^i, r_2^i)$ is either independent from the reference point pair (r_1^i, r_2^i) , or no offer is ever made that changes that reference point pair. Hence, $x_2(r_1^i, r_2^i)$ can be obtained as the sum of a geometric series. For instance, let $(r_1^k, r_2^k) \in P_{1,I}$ for all $k \geq i$. Then by Lemma B.1,

$$\begin{aligned} x_2(r_1^i, r_2^i) &= \delta - \delta^2 + \delta^2 x_2(r_1^{i+1}, r_2^{i+1}) \\ &= \delta(1 - \delta) + \delta^2(\delta - \delta^2 + \delta^2 x_2(r_1^{i+2}, r_2^{i+2})) \\ &= \delta(1 - \delta)(1 + \delta^2) + \delta^4 x_2(r_1^{i+2}, r_2^{i+2}) \\ &= \delta(1 - \delta)(1 + \delta^2 + \delta^4 + \dots) \\ &= \delta(1 - \delta) \times \frac{1}{1 - \delta^2} \\ &= x_2^{1,I}. \end{aligned}$$

The proof for P_ω where $\omega \neq 1, I$ and for Q_ω where $\omega \in \Omega$ is analogous to the proof for $P_{1, I}$. \square

The following lemma establishes some restrictions on how reference points can move through the sets P and Q .

Lemma B.4 *If $(r_1^i, r_2^i) \in P_{1, III}$, then $(r_1^i, r_2^{i+1}) \notin Q_{1, I}$.*

If $(r_1^i, r_2^i) \in P_{2, III}$, then $(r_1^i, r_2^{i+1}) \notin Q_{2, I}$.

If $(r_1^i, r_2^{i+1}) \in Q_{3, I}$, then $(r_1^{i+1}, r_2^{i+1}) \notin P_{1, I}$.

If $(r_1^i, r_2^{i+1}) \in Q_{3, II}$, then $(r_1^{i+1}, r_2^{i+1}) \notin P_{1, II}$.

Proof. Let $(r_1^i, r_2^i) \in P_{1, III}$ and assume $(r_1^i, r_2^{i+1}) \in Q_{1, I}$. Then from the definitions of $P_{1, III}$ and $Q_{1, I}$ we have

$$\begin{aligned} y_2(r_1^i, r_2^{i+1}) &> x_2(r_1^i, r_2^i) \geq r_2^i, \\ r_1^i &> x_1(r_1^{i+1}, r_2^{i+1}) > y_1(r_1^i, r_2^{i+1}), \text{ and} \\ r_2^{i+1} &> y_2(r_1^{i+1}, r_2^{i+2}) > x_2(r_1^{i+1}, r_2^{i+1}). \end{aligned}$$

Since $r_1^i > y_1(r_1^i, r_2^{i+1})$ and $r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})$ we have $r_1^{i+1} = r_1^i$ and $r_2^{i+2} = r_2^{i+1}$ by (9). From (U1) it follows that

$$y_2(r_1^i, r_2^{i+1}) = y_2(r_1^{i+1}, r_2^{i+2}).$$

Furthermore, since $x_2(r_1^i, r_2^i) \geq r_2^i$, (9) implies $r_2^{i+1} = x_2(r_1^i, r_2^i)$. Thus,

$$r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2}) = y_2(r_1^i, r_2^{i+1}) > x_2(r_1^i, r_2^i) = r_2^{i+1}.$$

This is a contradiction.

The proofs of the other statements are analogous. \square

B.2 Proof of Theorem 4.1

In this section we prove a series of lemmas which are used in the proof of Theorem 4.1.

Lemma B.5 *Let $\omega \in \{1, I, 1, II, 2, I, 2, II\}$. Then for all $(r_1^i, r_2^i) \in P_\omega$ we have $x(r_1^i, r_2^i) = x^\omega$. Similarly, for all $(r_1^i, r_2^{i+1}) \in Q_\omega$ we have $y(r_1^i, r_2^{i+1}) = y^\omega$.*

Proof. Let $(r_1^i, r_2^i) \in P_\omega$ where $\omega \in \{1, I, 1, II, 2, I, 2, II\}$. Then $r_1^i > y_1(r_1^i, r_2^{i+1})$ and $r_2^i > x_2(r_1^i, r_2^i)$. By (9) this implies $r_1^{i+1} = r_1^i$ and $r_2^{i+1} = r_2^i$. By (U1),

$$x_2(r_1^i, r_2^i) = x_2(r_1^{i+1}, r_2^{i+1}).$$

By Lemma B.2 $x_2(r_1^i, r_2^i) = x_2^\omega$. The case for Q_ω is similar. \square

Lemma B.6 *If $(r_1^i, r_2^{i+1}) \in Q_{3, I}$, then*

$$i. y_1(r_1^i, r_2^{i+1}) \geq y_1^{3,I}.$$

$$ii. y_1(r_1^i, r_2^{i+1}) \leq y_1^{3,I}.$$

Thus, $y_1(r_1^i, r_2^{i+1}) = y_1^{3,I}$.

Proof of Lemma B.6, part i. Let $(r_1^i, r_2^{i+1}) \in Q_{3,I}$. From the definition of $Q_{3,I}$ we have

$$\begin{aligned} x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i, \text{ and} \\ r_2^{i+1} &> y_2(r_1^{i+1}, r_2^{i+2}) > x_2(r_1^{i+1}, r_2^{i+1}). \end{aligned}$$

Assume $y_1(r_1^i, r_2^{i+1}) < y_1^{3,I}$. By Lemma B.2, $y_1(r_1^i, r_2^{i+1}) > y_1(r_1^{i+1}, r_2^{i+2})$. Since $y_1(r_1^i, r_2^{i+1}) \geq r_1^i$ we have from (9) that $r_1^{i+1} = y_1(r_1^i, r_2^{i+1})$. Hence, $r_1^{i+1} > y_1(r_1^{i+1}, r_2^{i+2})$. We have two possibilities:

- a. $r_1^{i+1} > x_1(r_1^{i+2}, r_2^{i+2}) > y_1(r_1^{i+1}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+1}) \in P_{1,I}$.
- b. $x_1(r_1^{i+2}, r_2^{i+2}) \geq r_1^{i+1} > y_1(r_1^{i+1}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+1}) \in P_{2,I}$.

By Lemma B.4, case a. can be ruled out. Hence, $(r_1^{i+1}, r_2^{i+1}) \in P_{2,I}$. By Lemma B.5, $x_2(r_1^{i+1}, r_2^{i+1}) = x_2^{2,I}$. Since $(r_1^i, r_2^{i+1}) \in Q_{3,I}$, Lemma B.1 implies

$$\begin{aligned} y_1(r_1^i, r_2^{i+1}) &= \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2(r_1^{i+1}, r_2^{i+1}) \\ &= \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2^{2,I} \\ &= \frac{\delta}{\mu_1} x_1^{2,I}. \end{aligned}$$

By (9), we have $r_1^{i+1} = y_1(r_1^i, r_2^{i+1})$. Note that $x_1^{2,I}$ is a function of r_1^{i+1} , and therefore of $y_1(r_1^i, r_2^{i+1})$. That is,

$$\begin{aligned} y_1(r_1^i, r_2^{i+1}) &= \frac{\delta}{\mu_1} \times \frac{(\mu_1 - \delta) + \delta^2 \lambda_1 r_1^{i+1}}{(1 + \lambda_1) - \delta^2} \\ &= \frac{\delta}{\mu_1} \times \frac{(\mu_1 - \delta) + \delta^2 \lambda_1 y_1(r_1^i, r_2^{i+1})}{(1 + \lambda_1) - \delta^2}. \end{aligned}$$

Solving for $y_1(r_1^i, r_2^{i+1})$ yields

$$\begin{aligned}
y_1(r_1^i, r_2^{i+1}) &= \frac{\delta}{\mu_1} \times \frac{\mu_1 - \delta}{1 + \lambda_1 - \delta^2 - \delta^2 \lambda_1 \frac{\delta}{\mu_1}} \\
&= \frac{\delta(\mu_1 - \delta)}{(1 + \lambda_1)\mu_1 - \delta^2\mu_1 - \delta^3\lambda_1} \\
&= \frac{\delta(\mu_1 - \delta)}{(1 + \lambda_1)\mu_1 - \delta^2 - \delta^2\lambda_1 + \delta^3\lambda_1 - \delta^3\lambda_1} \\
&= \frac{\delta(\mu_1 - \delta)}{(1 + \lambda_1)(\mu_1 - \delta^2)} \\
&= \frac{\delta}{1 + \lambda_1 + \delta} \\
&= y_1^{3,I},
\end{aligned}$$

where we used the fact that

$$\begin{aligned}
(1 + \lambda_1)(\mu_1 - \delta^2) &= (1 + \lambda_1)(1 + \lambda_1(1 - \delta) - \delta^2) \\
&= (1 + \lambda_1)((1 + \delta)(1 - \delta) + \lambda_1(1 - \delta)) \\
&= (1 + \lambda_1)(1 - \delta)(1 + \lambda_1 + \delta) \\
&= (1 - \delta + \lambda_1(1 - \delta))(1 + \lambda_1 + \delta) \\
&= (\mu_1 - \delta)(1 + \lambda_1 + \delta).
\end{aligned}$$

Observe that $y_1(r_1^i, r_2^{i+1}) = y_1^{3,I}$ is contradicting $y_1(r_1^i, r_2^{i+1}) < y_1^{3,I}$, our initial assumption. \square

Before we can prove Part *ii*, we must make a similar argument for $Q_{3,II}$.

Lemma B.7 *If $(r_1^i, r_2^{i+1}) \in Q_{3,II}$, then*

i. $y_1(r_1^i, r_2^{i+1}) \geq y_1^{3,II}$.

ii. $y_1(r_1^i, r_2^{i+1}) \leq y_1^{3,II}$.

Thus, $y_1(r_1^i, r_2^{i+1}) = y_1^{3,II}$.

Proof of Lemma B.7, Part *i*. Let $(r_1^i, r_2^{i+1}) \in Q_{3,II}$. From the definition of $Q_{3,II}$ it follows that

$$\begin{aligned}
x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i, \text{ and} \\
y_2(r_1^{i+1}, r_2^{i+1}) &\geq r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1}).
\end{aligned}$$

Assume $y_1(r_1^i, r_2^{i+1}) < y_1^{3,II}$. Thus, $y_1(r_1^i, r_2^{i+1}) > y_1(r_1^{i+1}, r_2^{i+2})$ by Lemma B.2. Since $y_1(r_1^i, r_2^{i+1}) \geq r_1^i$, we have $r_1^{i+1} = y_1(r_1^i, r_2^{i+1})$ by (9). Hence, $r_1^{i+1} > y_1(r_1^{i+1}, r_2^{i+2})$, leaving two possibilities:

a. $r_1^{i+1} \geq x_1(r_1^{i+2}, r_2^{i+2}) > y_1(r_1^{i+1}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+1}) \in P_{1,II}$.

b. $x_1(r_1^{i+2}, r_2^{i+2}) > r_1^{i+1} > y_1(r_1^{i+1}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+1}) \in P_{2,\text{II}}$.

Case a. is ruled out by Lemma B.4. Hence, $(r_1^{i+1}, r_2^{i+1}) \in P_{2,\text{II}}$, and thus by Lemma B.5, $x_2(r_1^{i+1}, r_2^{i+1}) = x_2^{2,\text{II}}$. Since $(r_1^i, r_2^{i+1}) \in Q_{3,\text{II}}$, we have from Lemma B.1 that

$$\begin{aligned} y_1(r_1^i, r_2^{i+1}) &= \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2(r_1^{i+1}, r_2^{i+1}) \\ &= \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2^{2,\text{II}} \\ &= \frac{\delta}{\mu_1} x_1^{2,\text{II}}. \end{aligned}$$

Note that $y_1(r_1^i, r_2^{i+1}) = r_1^{i+1}$ and that $x_1^{2,\text{II}}$ is a function of r_1^{i+1} . That is,

$$\begin{aligned} y_1(r_1^i, r_2^{i+1}) &= \frac{(1 + \lambda_1)((\mu_2 - \delta) + \delta\lambda_2(1 - r_2^{i+1})) + \delta^2\lambda_1 r_1^{i+1}}{(1 + \lambda_1)(1 + \lambda_2) - \delta^2} \\ &= \frac{(1 + \lambda_1)((\mu_2 - \delta) + \delta\lambda_2(1 - r_2^{i+1})) + \delta^2\lambda_1 y_1(r_1^i, r_2^{i+1})}{(1 + \lambda_1)(1 + \lambda_2) - \delta^2}. \end{aligned}$$

Solving for $y_1(r_1^i, r_2^{i+1})$ yields

$$\begin{aligned} y_1(r_1^i, r_2^{i+1}) &= \frac{\delta}{\mu_1} \times \frac{(1 + \lambda_1)(\mu_2 - \delta + \delta\lambda_2(1 - r_2^{i+1}))}{(1 + \lambda_1)(1 + \lambda_2) - \delta^2 - \delta^2\lambda_1 \frac{\delta}{\mu_1}} \\ &= \frac{\delta(1 + \lambda_1)(\mu_2 - \delta + \delta\lambda_2(1 - r_2^{i+1}))}{(1 + \lambda_1)(1 + \lambda_2)\mu_1 - \delta^2\mu_1 - \delta^3\lambda_1} \\ &= \frac{\delta(1 + \lambda_1)(\mu_2 - \delta + \delta\lambda_2(1 - r_2^{i+1}))}{(1 + \lambda_1)(1 + \lambda_2)\mu_1 - \delta^2 - \delta^2\lambda_1 + \delta^3\lambda_1 - \delta^3\lambda_1} \\ &= \frac{\delta(1 + \lambda_1)(\mu_2 - \delta + \delta\lambda_2(1 - r_2^{i+1}))}{(1 + \lambda_1)((1 + \lambda_2)\mu_1 - \delta^2)} \\ &= \frac{\delta(\mu_2 - \delta + \delta\lambda_2(1 - r_2^{i+1}))}{(1 + \lambda_2)\mu_1 - \delta^2} \\ &= y_1^{3,\text{II}}. \end{aligned}$$

This contradicts $y_1(r_1^i, r_2^{i+1}) < y_1^{3,\text{II}}$. □

This result allows us to prove the second part of Lemma B.6.

Proof of Lemma B.6, Part ii. Let $(r_1^i, r_2^{i+1}) \in Q_{3,\text{I}}$. By the definition of $Q_{3,\text{I}}$ we have

$$\begin{aligned} x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i, \text{ and} \\ r_2^{i+1} &> y_2(r_1^{i+1}, r_2^{i+2}) > x_2(r_1^{i+1}, r_2^{i+1}). \end{aligned}$$

Assume $y_1(r_1^i, r_2^{i+1}) > y_1^{3,I}$. Then by Lemma B.2 we have $y_1(r_1^{i+1}, r_2^{i+2}) > y_1(r_1^i, r_2^{i+1})$. Observe that $y_1(r_1^i, r_2^{i+1}) \geq r_1^i$, and thus by (9) $y_1(r_1^i, r_2^{i+1}) = r_1^{i+1}$. Hence, $y_1(r_1^{i+1}, r_2^{i+2}) > r_1^{i+1}$, implying $(r_1^{i+1}, r_2^{i+2}) \in P_{3,I}$. Then from Lemma B.1, $y_1(r_1^i, r_2^{i+1}) > y_1^{3,I}$ and $y_1(r_1^{i+1}, r_2^{i+2}) > y_1(r_1^i, r_2^{i+1})$, we have

$$x_2(r_1^{i+1}, r_2^{i+1}) = \delta - \delta y_1(r_1^{i+1}, r_2^{i+2}) < \delta - \delta y_1(r_1^i, r_2^{i+1}) < \delta - \delta y_1^{3,I} = x_2^{3,I}.$$

By Lemma B.2 this implies $x_2(r_1^{i+1}, r_2^{i+1}) > x_2(r_1^{i+2}, r_2^{i+2})$. Observe that, by (9), $r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})$ implies $r_2^{i+2} = r_2^{i+1}$, and thus

$$r_2^{i+2} = r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1}) > x_2(r_1^{i+2}, r_2^{i+2}).$$

This leaves two possibilities.

- a. $r_2^{i+2} > y_2(r_1^{i+2}, r_2^{i+3}) > x_2(r_1^{i+2}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,I}$.
- b. $y_2(r_1^{i+2}, r_2^{i+3}) \geq r_2^{i+2} > x_2(r_1^{i+2}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,II}$.

Take case b., i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,II}$. Then

$$y_2(r_1^{i+2}, r_2^{i+3}) \geq r_2^{i+2} = r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2}),$$

which implies $y_1(r_1^{i+1}, r_2^{i+2}) > y_1(r_1^{i+2}, r_2^{i+3})$. By Lemma B.2 we have $y_1(r_1^{i+1}, r_2^{i+2}) < y_1^{3,II}$. This contradicts part *i*. of Lemma B.7.

Hence, case a. must hold, i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,I}$. Note that $x_2(r_1^{i+2}, r_2^{i+2}) < x_2(r_1^{i+1}, r_2^{i+1}) < x_2^{3,I}$. Then

$$y_1(r_1^{i+1}, r_2^{i+2}) = \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2(r_1^{i+2}, r_2^{i+2}) > \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2^{3,I} = y_1^{3,I}.$$

Thus, if $(r_1^i, r_2^{i+1}) \in Q_{3,I}$ and $y_1(r_1^i, r_2^{i+1}) > y_1^{3,I}$, then $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,I}$ and $y_1(r_1^{i+1}, r_2^{i+2}) > y_1^{3,I}$. Thus, for all $k \geq i$ we have $(r_1^k, r_2^{k+1}) \in Q_{3,I}$. Then by Lemma B.3 we have that $y_1(r_1^i, r_2^{i+1}) = y_1^{3,I}$. This contradicts $y_1(r_1^i, r_2^{i+1}) > y_1^{3,I}$. \square

We can now also complete the proof of Lemma B.7.

Proof of Lemma B.7, Part *ii*. Let $(r_1^i, r_2^{i+1}) \in Q_{3,II}$. By definition of $Q_{3,II}$ we have

$$\begin{aligned} x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i \\ y_2(r_1^{i+1}, r_2^{i+2}) &\geq r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1}). \end{aligned}$$

Assume $y_1(r_1^i, r_2^{i+1}) > y_1^{3,II}$. Then by Lemma B.2 we have $y_1(r_1^i, r_2^{i+1}) < y_1(r_1^{i+1}, r_2^{i+2})$. Since $y_1(r_1^i, r_2^{i+1}) \geq r_1^i$, (9) says that $r_1^{i+1} = y_1(r_1^i, r_2^{i+1})$. Hence, $y_1(r_1^{i+1}, r_2^{i+2}) > r_1^{i+1}$, which implies $(r_1^{i+1}, r_2^{i+1}) \in P_{3,II}$. From Lemma B.1,

$y_1(r_1^i, r_2^{i+1}) < y_1(r_1^{i+1}, r_2^{i+2})$, $y_1^{3,\text{II}} < y_1(r_1^i, r_2^{i+1})$, and the fact that $r_2^{i+2} = \max\{r_2^{i+1}, x_2(r_1^{i+1}, r_2^{i+1})\} = r_2^{i+1}$, we have

$$\begin{aligned} x_2(r_1^{i+1}, r_2^{i+1}) &= \frac{\delta(1 + \lambda_2 r_2^{i+2})}{1 + \lambda_2} - \frac{\delta}{1 + \lambda_2} y_1(r_1^{i+1}, r_2^{i+2}) \\ &< \frac{\delta(1 + \lambda_2 r_2^{i+2})}{1 + \lambda_2} - \frac{\delta}{1 + \lambda_2} y_1(r_1^i, r_2^{i+1}) \\ &< \frac{\delta(1 + \lambda_2 r_2^{i+2})}{1 + \lambda_2} - \frac{\delta}{1 + \lambda_2} y_1^{3,\text{II}} \\ &= \frac{\delta(1 + \lambda_2 r_2^{i+1})}{1 + \lambda_2} - \frac{\delta}{1 + \lambda_2} y_1^{3,\text{II}} \\ &= x_2^{3,\text{II}}. \end{aligned}$$

By Lemma B.2 this implies $x_2(r_1^{i+1}, r_2^{i+1}) > x_2(r_1^{i+2}, r_2^{i+2})$. Since $r_2^{i+2} = r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})$, this implies $r_2^{i+2} > x_2(r_1^{i+2}, r_2^{i+2})$. Then there are two possibilities:

- a. $r_2^{i+2} > y_2(r_1^{i+2}, r_2^{i+3}) > x_2(r_1^{i+2}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,\text{I}}$.
- b. $y_2(r_1^{i+2}, r_2^{i+3}) \geq r_2^{i+2} > x_2(r_1^{i+2}, r_2^{i+2})$, i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,\text{II}}$.

Take case a., i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,\text{I}}$. Observe that

$$y_2(r_1^{i+1}, r_2^{i+2}) \geq r_2^{i+1} = r_2^{i+2} > y_2(r_1^{i+2}, r_2^{i+3}),$$

and thus $y_1(r_1^{i+1}, r_2^{i+2}) < y_1(r_1^{i+2}, r_2^{i+3})$. Lemma B.2 now implies $y_1(r_1^{i+1}, r_2^{i+2}) > y_1^{3,\text{I}}$. This contradicts part *i.* of Lemma B.6.

Hence, case b. must hold, i.e. $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,\text{II}}$. Since $x_2(r_1^{i+1}, r_2^{i+1}) > x_2^{3,\text{II}}$ and $x_2(r_1^{i+2}, r_2^{i+2}) > x_2(r_1^{i+1}, r_2^{i+1})$, we have from Lemma B.1

$$y_1(r_1^{i+1}, r_2^{i+2}) = \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2(r_1^{i+2}, r_2^{i+2}) > \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2^{3,\text{II}} = y_1^{3,\text{II}}.$$

Thus, $(r_1^i, r_2^{i+1}) \in Q_{3,\text{II}}$ and $y_1(r_1^i, r_2^{i+1}) > y_1^{3,\text{II}}$ implies $(r_1^{i+1}, r_2^{i+2}) \in Q_{3,\text{II}}$ and $y_1(r_1^{i+1}, r_2^{i+2}) > y_1^{3,\text{II}}$. Then, for all $k \geq i$ we have $(r_1^k, r_2^{k+1}) \in Q_{3,\text{II}}$, which by Lemma B.3 implies $y_1(r_1^i, r_2^{i+1}) = y_1^{3,\text{II}}$. This contradicts $y_1(r_1^i, r_2^{i+1}) > y_1^{3,\text{II}}$. \square

Similar results hold in $P_{2,\text{III}}$ and $P_{1,\text{III}}$.

Lemma B.8 *If $(r_1^i, r_2^i) \in P_{1,\text{III}}$, then $x_2(r_1^i, r_2^i) = x_2^{1,\text{III}}$.*

Lemma B.9 *If $(r_1^i, r_2^i) \in P_{2,\text{III}}$, then $x_2(r_1^i, r_2^i) = x_2^{2,\text{III}}$.*

The proofs of these lemmas are analogous to the proofs of Lemmas B.6 and B.7 respectively.

Lemma B.10 *If $(r_1^i, r_2^i) \in P_{3,\text{III}}$, then*

$$i. x_2(r_1^i, r_2^i) \geq x_2^{3,III}.$$

$$ii. x_2(r_1^i, r_2^i) \leq x_2^{3,III}.$$

Thus, $x_2(r_1^i, r_2^i) = x_2^{3,III}$.

Proof of Lemma B.10, Part i. Let $(r_1^i, r_2^i) \in P_{3,III}$. By the definition of $P_{3,III}$ we have

$$\begin{aligned} y_2(r_1^i, r_2^{i+1}) &> x_2(r_1^i, r_2^i) \geq r_2^i, \\ x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i. \end{aligned}$$

Assume $x_2(r_1^i, r_2^i) < x_2^{3,III}$. Lemma B.2 implies $x_2(r_1^i, r_2^i) > x_2(r_1^{i+1}, r_2^{i+1})$. Since $x_2(r_1^i, r_2^i) \geq r_2^i$, we have from (9) that $r_2^{i+1} = x_2(r_1^i, r_2^i)$. Hence, $r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})$. Then there are two possibilities:

$$a. r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2}) > x_2(r_1^{i+1}, r_2^{i+1}), \text{ i.e. } (r_1^i, r_2^{i+1}) \in Q_{3,I}.$$

$$b. y_2(r_1^{i+1}, r_2^{i+2}) \geq r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1}), \text{ i.e. } (r_1^i, r_2^{i+1}) \in Q_{3,II}.$$

Take case a., i.e. $(r_1^i, r_2^{i+1}) \in Q_{3,I}$. From Lemma B.6 we have $y_1(r_1^i, r_2^{i+1}) = y_1^{3,I}$. From Lemma B.1 it follows that $x_2(r_1^i, r_2^i) = \frac{\delta}{\mu_2} - \frac{\delta}{\mu_2} y_1^{3,I} = \frac{\delta}{\mu_2} y_2^{3,I}$. Observe that $\mu_2(\mu_1 - \delta^2) < \mu_1\mu_2 - \delta^2$. Thus,

$$\begin{aligned} y_2^{3,I} &= \frac{1 + \lambda_1}{1 + \lambda_1 + \delta} = \frac{(1 + \lambda_1)(1 - \delta)}{(1 + \lambda_1 + \delta)(1 - \delta)} = \frac{(1 - \delta + \lambda_1(1 - \delta))}{1 - \delta + \lambda_1(1 - \delta) + \delta - \delta^2} \\ &= \frac{\mu_2}{\mu_2} \times \frac{(\mu_1 - \delta)}{\mu_1 - \delta^2} > \frac{\mu_2(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} = y_2^{3,III}. \end{aligned} \quad (11)$$

Then $x_2(r_1^i, r_2^i) > \frac{\delta}{\mu_2} y_2^{3,III} = x_2^{3,III}$, contradicting the initial assumption $x_2(r_1^i, r_2^i) < x_2^{3,III}$.

Take case b., i.e., $(r_1^i, r_2^{i+1}) \in Q_{3,II}$. By Lemma B.1, $x_2(r_1^i, r_2^i) = \frac{\delta}{\mu_2} y_2^{3,II}$. Note that $r_2^{i+1} = x_2(r_1^i, r_2^i)$ by (9), and that $y_2^{3,II}$ is a function of r_2^{i+1} . That is,

$$\begin{aligned} x_2(r_1^i, r_2^i) &= \frac{\delta}{\mu_2} \times \frac{(\mu_1 - \delta)(1 + \lambda_2) + \delta^2 \lambda_2 r_2^{i+1}}{\mu_1(1 + \lambda_2) - \delta^2} \\ &= \frac{\delta}{\mu_2} \times \frac{(\mu_1 - \delta)(1 + \lambda_2) + \delta^2 \lambda_2 x_2(r_1^i, r_2^i)}{\mu_1(1 + \lambda_2) - \delta^2}. \end{aligned}$$

Solving for $x_2(r_1^i, r_2^i)$ yields

$$\begin{aligned} x_2(r_1^i, r_2^i) &= \frac{\delta}{\mu_2} \times \frac{(\mu_1 - \delta)(1 + \lambda_2)}{\mu_1(1 + \lambda_2) - \delta^2 - \delta^2 \lambda_2 \frac{\delta}{\mu_2}} = \frac{\delta(\mu_1 - \delta)(1 + \lambda_2)}{\mu_1\mu_2(1 + \lambda_2) - \delta^2\mu_2 - \delta^3\lambda_2} \\ &= \frac{\delta(\mu_1 - \delta)(1 + \lambda_2)}{\mu_1\mu_2(1 + \lambda_2) - \delta^2 - \delta^2\lambda_2 + \delta^3\lambda_2 - \delta^3\lambda_2} = \frac{\delta(\mu_1 - \delta)}{\mu_1\mu_2 - \delta^2} = x_2^{3,III}. \end{aligned}$$

This contradicts $x_2(r_1^i, r_2^i) < x_2^{3,III}$. \square

Before we can complete the proof of Lemma B.10, we need a similar result for $Q_{3,III}$.

Lemma B.11 *If $(r_1^i, r_2^{i+1}) \in Q_{3,III}$, then*

$$i. \quad y_1(r_1^i, r_2^{i+1}) \geq y_1^{3,III},$$

$$ii. \quad y_1(r_1^i, r_2^{i+1}) \leq y_1^{3,III}.$$

Thus, $y_1(r_1^i, r_2^{i+1}) = y_1^{3,III}$.

Proof of Lemma B.11, Part i. This is analogous to the proof of Lemma B.10 Part i. \square

We continue with the second part of Lemma B.10.

Proof of Lemma B.10, Part ii. Let $(r_1^i, r_2^i) \in P_{3,III}$. By definition of $P_{3,III}$ we have

$$\begin{aligned} y_2(r_1^i, r_2^{i+1}) &> x_2(r_1^i, r_2^i) \geq r_2^i, \text{ and} \\ x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i. \end{aligned}$$

Assume $x_2(r_1^i, r_2^i) > x_2^{3,III}$. Then by Lemma B.2 $x_2(r_1^i, r_2^i) < x_2(r_1^{i+1}, r_2^{i+1})$. Since $x_2(r_1^i, r_2^i) \geq r_2^i$, we have $r_2^{i+1} = x_2(r_1^i, r_2^i)$ by (9). Thus, $r_2^{i+1} < x_2(r_1^{i+1}, r_2^{i+1})$, implying $(r_1^i, r_2^{i+1}) \in Q_{3,III}$. By the first part of Lemma B.11 this implies that $y_1(r_1^i, r_2^{i+1}) \geq y_1^{3,III}$. Then from Lemma B.1 and the construction of $x^{3,III}$ we obtain

$$x_2(r_1^i, r_2^i) = \frac{\delta}{\mu_2} - \frac{\delta}{\mu_2} y_1(r_1^i, r_2^{i+1}) \leq \frac{\delta}{\mu_2} - \frac{\delta}{\mu_2} y_1^{3,III} = x_2^{3,III}.$$

This contradicts $x_2(r_1^i, r_2^i) > x_2^{3,III}$. \square

In a similar way we can complete the proof of Lemma B.11.

Proof of Lemma B.11, Part ii. Analogous to the proof of Lemma B.10, Part ii. \square

redWe can now obtain the offers made in the sets $P_{3,I}$ and $P_{3,II}$.

Lemma B.12 *If $(r_1^i, r_2^i) \in P_{3,I}$ then $x_2(r_1^i, r_2^i) = x_2^{3,I}$.*

Proof. Let $(r_1^i, r_2^i) \in P_{3,I}$. By definition of $P_{3,I}$ we have

$$\begin{aligned} r_2^i &> y_2(r_1^i, r_2^{i+1}) > x_2(r_1^i, r_2^i), \text{ and} \\ x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i. \end{aligned}$$

Assume $x_2(r_1^i, r_2^i) \neq x_2^{3,I}$. We have three mutually exclusive and exhaustive cases.

- $(r_1^i, r_2^{i+1}) \in Q_{3,I}$: Note that $(r_1^i, r_2^i) \in P_{3,I}$ by Lemma B.1 implies $x_2(r_1^i, r_2^i) = \delta - \delta y_1(r_1^i, r_2^{i+1}) = \delta y_2(r_1^i, r_2^{i+1})$. Then by Lemma B.6 and the construction of $x^{3,I}$ this implies $x_2(r_1^i, r_2^i) = \delta y_2^{3,I} = x_2^{3,I}$, contradicting $x_2(r_1^i, r_2^i) \neq x_2^{3,I}$.

- $(r_1^i, r_2^{i+1}) \in Q_{3,II}$: Since $(r_1^i, r_2^i) \in P_{3,I}$ and $(r_1^i, r_2^{i+1}) \in Q_{3,II}$ we have

$$y_2(r_1^{i+1}, r_2^{i+2}) \geq r_2^{i+1} = \max\{r_2^i, x_2(r_1^i, r_2^i)\} = r_2^i > y_2(r_1^i, r_2^{i+1}).$$

However, $y_2(r_1^i, r_2^{i+1}) = y_2^{3,II}$ by Lemma B.7, and this implies $y_2(r_1^i, r_2^{i+1}) = y_2(r_1^{i+1}, r_2^{i+2})$ by Lemma B.2. This contradicts the above.

- $(r_1^i, r_2^{i+1}) \in Q_{3,III}$: By Lemma B.1, Lemma B.11, inequality (11), and the construction of $x^{3,I}$, we have

$$x_2(r_1^i, r_2^i) = \delta - \delta y_1(r_1^i, r_2^{i+1}) = \delta y_2^{3,III} < \delta y_2^{3,I} = x_2^{3,I}.$$

By Lemma B.2 this implies $x_2(r_1^i, r_2^i) > x_2(r_1^{i+1}, r_2^{i+1})$. Since $r_2^i > x_2(r_1^i, r_2^i)$ we have by (9) that $r_2^{i+1} = r_2^i$, implying $r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})$. This implies $(r_1^i, r_2^{i+1}) \notin Q_{3,III}$, a contradiction.

It follows that $x_2(r_1^i, r_2^i) = x_2^{3,I}$. □

Lemma B.13 *If $(r_1^i, r_2^i) \in P_{3,II}$ then $x_2(r_1^i, r_2^i) = x_2^{3,II}$.*

Proof. Let $(r_1^i, r_2^i) \in P_{3,II}$. By definition of $P_{3,II}$ we have

$$\begin{aligned} y_2(r_1^i, r_2^{i+1}) &\geq r_2^i > x_2(r_1^i, r_2^i) \\ x_1(r_1^{i+1}, r_2^{i+1}) &> y_1(r_1^i, r_2^{i+1}) \geq r_1^i. \end{aligned}$$

Assume $x_2(r_1^i, r_2^i) \neq x_2^{3,II}$. Then we distinguish three mutually exclusive and exhaustive cases.

- $(r_1^i, r_2^{i+1}) \in Q_{3,I}$: Since $r_2^i > x_2(r_1^i, r_2^i)$ we have from (9) that $r_2^i = r_2^{i+1}$. Hence,

$$y_2(r_1^i, r_2^{i+1}) \geq r_2^i = r_2^{i+1} > y_2(r_1^{i+1}, r_2^{i+2}).$$

However, by Lemma B.6 we have $y_2(r_1^i, r_2^{i+1}) = y_2^{3,I}$, which by Lemma B.2 implies $y_2(r_1^i, r_2^{i+1}) = y_2(r_1^{i+1}, r_2^{i+2})$. This contradicts the above.

- $(r_1^i, r_2^{i+1}) \in Q_{3,II}$: By Lemma B.7 we have $y_1(r_1^i, r_2^{i+1}) = y_1^{3,II}$. By Lemma B.1 and the construction of $x^{3,II}$, we then have

$$\begin{aligned} x_2(r_1^i, r_2^i) &= \frac{\delta(1 + \lambda_2 r_2^{i+1})}{1 + \lambda_2} - \frac{\delta y_1(r_1^i, r_2^{i+1})}{1 + \lambda_2} \\ &= \frac{\delta(1 + \lambda_2 r_2^{i+1})}{1 + \lambda_2} - \frac{\delta y_1^{3,II}}{1 + \lambda_2} \\ &= \frac{\delta(y_2^{3,II} + \lambda_2 r_2^{i+1})}{1 + \lambda_2} \\ &= x_2^{3,II}, \end{aligned}$$

contradicting $x_2(r_1^i, r_2^i) \neq x_2^{3,II}$.

- $(r_1^i, r_2^{i+1}) \in Q_{3,III}$: By Lemma B.11 and the definition of $y_1^{3,III}$ this implies

$$y_1(r_1^i, r_2^{i+1}) = y_1^{3,III} = \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2^{3,III}.$$

By Lemma B.1 we have

$$y_1(r_1^i, r_2^{i+1}) = \frac{\delta}{\mu_1} - \frac{\delta}{\mu_1} x_2(r_1^{i+1}, r_2^{i+1}).$$

Hence, $x_2(r_1^{i+1}, r_2^{i+1}) = x_2^{3,III}$.

Since $r_2^i > x_2(r_1^i, r_2^i)$, we have from (9) that $r_2^{i+1} = r_2^i$. Then

$$x_2(r_1^{i+1}, r_2^{i+1}) \geq r_2^{i+1} = r_2^i > x_2(r_1^i, r_2^i).$$

By Lemma B.2 this implies $x_2(r_1^i, r_2^i) > x_2^{3,III}$. Since $r_2^i > x_2(r_1^i, r_2^i)$ and $r_2^i = r_2^{i+1}$ this implies $r_2^{i+1} > x_2^{3,III}$. That is,

$$r_2^{i+1} > \frac{\delta(\mu_1 - \delta + \lambda_2 r_2^{i+1} \mu_1)}{\mu_1(1 + \lambda_2) - \delta^2}.$$

This is equivalent to $r_2^{i+1} > \frac{\delta(\mu_1 - \delta)}{\mu_1 \mu_2 - \delta^2} = x_2^{3,III}$. Hence, $r_2^{i+1} > x_2(r_1^{i+1}, r_2^{i+1})$, which implies $(r_1^i, r_2^{i+1}) \notin Q_{3,III}$. This is a contradiction.

It follows that $x_2(r_1^i, r_2^i) = x_2^{3,III}$. □

Similar results hold for $Q_{2,III}$ and $Q_{1,III}$.

Lemma B.14 *If $(r_1^i, r_2^{i+1}) \in Q_{1,III}$ then $y_1(r_1^i, r_2^{i+1}) = y_1^{1,III}$.*

Lemma B.15 *If $(r_1^i, r_2^{i+1}) \in Q_{2,III}$ then $y_1(r_1^i, r_2^{i+1}) = y_1^{2,III}$.*

Proofs are analogous to Lemma B.12 resp. B.13. We now prove the main result of this section.

Proof of Theorem 4.1. Let $(r_1^i, r_2^i) = P_{1,I}$. By definition we have $r_2^i > y_2(r_1^i, r_2^{i+1})$ and $r_1^i > x_1(r_1^i, r_2^i)$. By Lemma B.5 this implies $r_2^i > y_2^{1,I}$ and $r_1^i > x_1^{1,I}$. Hence, $(r_1^i, r_2^i) \in X_{1,I}$, from which it follows that $P_{1,I} \subseteq X_{1,I}$.

Similarly, by Lemmas B.5, B.8, B.9, B.10, B.12, and B.13, $(r_1^i, r_2^i) \in P_\omega$ implies $(r_1^i, r_2^i) \in X_\omega$ for each $\omega \in \Omega$. Hence, $P_\omega \subseteq X_\omega$ for each $\omega \in \Omega$. Since the sets P_ω are mutually exclusive and exhaustive, this implies

$$P_\omega = X_\omega \text{ for each } \omega \in \Omega.$$

Hence, at times $t \in T_{odd}$ player 1's unique SPE strategy is to propose x^ω if $(r_1, r_2) \in X_\omega$. That is, to play strategy f . Analogously, we have

$$Q_\omega = X_\omega \text{ for each } \omega \in \Omega,$$

such that player 2's unique SPE strategy at times $t \in T_{\text{even}}$ is to propose y^ω if $(r_1, r_2) \in X_\omega$, i.e. to follow strategy \hat{g} .

It follows from part B. of the proof of Theorem 3.1 that player 1's unique optimal strategy at times $t \in T_{\text{even}}$ is to accept proposals which are at or above 2's SPE proposal, and to reject those which are not. (Rejecting player 2's proposal violates immediate acceptance.) That is, player 1's unique SPE strategy at times $t \in T_{\text{even}}$ is \hat{f} . Similarly, player 2's unique optimal strategy at times $t \in T_{\text{odd}}$ is to accept and reject proposals according to the strategy \hat{g} .

Thus, the strategy profile (\hat{f}, \hat{g}) is the unique SPE satisfying (U1)–(U3). \square

C Concave utility functions

Let player i 's preference for pie, $i = 1, 2$, be represented by a strictly increasing, concave von Neumann-Morgenstern utility function $v_i : [0, 1] \rightarrow \mathbb{R}$. Since such a utility function is unique up to an affine transformation, we may assume that $v_i(0) = 0$ and $v_i(1) = 1$. Furthermore, define

$$\tilde{Z} := \{(v_1(z_1), v_2(z_2)) \mid z \in Z\}.$$

In this more general setting, each finite path in \bar{H} is associated with a *utility* outcome. That is, for all $h \in \bar{H} \setminus H^\infty$, we say that player i , $i = 1, 2$, obtains $v_i(\xi_i(h))$, rather than $\xi_i(h)$. Note that due to rescaling, both players still attach a zero utility payoff to a disagreement path, and a utility payoff of one to agreement on a partition that gives them the whole pie. The loss aversion transformation w is then applied to the utility outcome $v_i(\xi_i(h))$. The reference point r_i used in this transformation, is defined as the utility associated with the highest share player i has rejected in the past. Like before, we have

- a. $r_1 > x_1 > y_1$: $\delta_1 x_1 = y_1$.
- b. $x_1 \geq r_1 > y_1$: $\delta_1 x_1 = (1 + \lambda_1)y_1 - \delta_1 \lambda_1 r_1$.
- c. $x_1 > y_1 \geq r_1$: $\delta_1 x_1 = \mu_1 y_1$.

and

- i. $r_2 > y_2 > x_2$: $\delta_2 y_2 = x_2$.
- ii. $y_2 \geq r_2 > x_2$: $\delta_2 y_2 = (1 + \lambda_2)x_2 - \delta_2 \lambda_2 r_2$.
- iii. $y_2 > x_2 \geq r_2$: $\delta_2 y_2 = \mu_2 x_2$.

where $x, y \in \tilde{Z}$. These equations again define a ninefold partition of the set of all possible reference point pairs, similar to the one depicted in Figure 1. Define $\tilde{\Omega} := \{\text{a.i.}, \dots, \text{c.iii.}\}$, and denote the associated sets again by X_ω , $\omega \in \tilde{\Omega}$. It can be shown that the system of equations yields a unique solution (x^ω, y^ω) in

$\tilde{Z} \times \tilde{Z}$ for each $\omega \in \tilde{\Omega}$. Let $h^t = (h^{t-1}, z)$ with $z \in \tilde{Z}$. Then for all $\omega \in \tilde{\Omega}$, define the strategy pair (\check{f}, \check{g}) for $t \in T_{odd}$ by

$$\check{f}^t(h^{t-1}) = x^\omega, \quad \check{g}^t(h^t) = \begin{cases} Y & \text{if } z_2 \geq x_2^\omega, \\ N & \text{otherwise.} \end{cases}, \quad \text{if } (r_1(h^{t-1}), r_2(h^{t-1})) \in X_\omega,$$

and for $t \in T_{even}$ by

$$\check{f}^t(h^t) = \begin{cases} Y & \text{if } z_1 \geq y_1^\omega, \\ N & \text{otherwise} \end{cases}, \quad \check{g}^t(h^{t-1}) = y^\omega, \quad \text{if } (r_1(h^{t-1}), r_2(h^{t-1})) \in X_\omega.$$

We obtain the following result.

Theorem C.1 *The strategy profile (\check{f}, \check{g}) is an SPE.*

Proof. Observe that Lemma A.1 continues to hold if players' utility functions are concave. Hence, the game is still continuous at infinity. The proof of Theorem C.1 is then analogous to that of Theorem 3.1. \square

It follows that the partition obtained at the beginning of the game, where reference points are zero, is given by $x^{c.iii.}$. Henceforth, we drop the superscript c.iii., and write the solution as x . Note that we may again generalize the above results to the case where the time lapse between proposals is Δ . Thus, we have

$$\psi(x_1) = x_2 = \frac{\delta_2^\Delta}{\mu_2} y_2^{c.iii.} = \frac{\delta_2^\Delta}{\mu_2} \psi(y_1^{c.iii.}) = \frac{\delta_2^\Delta}{\mu_2} \psi\left(\frac{\delta_1^\Delta}{\mu_1} x_1\right)$$

where $\mu_i = 1 + \lambda_i(1 - \delta_i^\Delta)$. Then by extending Proposition 4 from Binmore *et al.* (1986), we generalize the result that the equilibrium partition converges to an asymmetric Nash bargaining solution partition as Δ goes to zero, to the case where players have concave utility functions.

Let z^N be the asymmetric Nash solution

$$z^N = \arg \max_{z \in \tilde{Z}} z_1^\alpha z_2^{1-\alpha},$$

where $\alpha = \frac{(1+\lambda_2) \log \delta_2}{(1+\lambda_1) \log \delta_1 + (1+\lambda_2) \log \delta_2}$. Observe that z^N is uniquely defined by

- $z_2^N = \psi(z_1^N)$, and
- $|\psi'_-(z_1^N)| \leq \frac{\alpha}{1-\alpha} \frac{z_2^N}{z_1^N} \leq |\psi'_+(z_1^N)|$,

where ψ'_- and ψ'_+ are the left- resp. righthand side derivatives of ψ . Let $x \rightarrow \bar{x}$ as $\Delta \rightarrow 0$. Since ψ is continuous on $[0, 1]$, we have that $\bar{x}_2 = \psi(\bar{x}_1)$. Furthermore, since ψ is decreasing and concave, we have

$$|\psi'_-(\bar{x}_1)| \leq \lim_{\Delta \rightarrow 0} \left| \frac{\psi(x_1) - \psi\left(\frac{\delta_1^\Delta}{\mu_1} x_1\right)}{x_1 - \frac{\delta_1^\Delta}{\mu_1} x_1} \right| \leq |\psi'_+(\bar{x}_1)|,$$

and, using l'Hôpital's rule,

$$\begin{aligned}
\lim_{\Delta \rightarrow 0} \left| \frac{\psi(x_1) - \psi\left(\frac{\delta_1^\Delta}{\mu_1} x_1\right)}{x_1 - \frac{\delta_1^\Delta}{\mu_1} x_1} \right| &= \lim_{\Delta \rightarrow 0} \left| \frac{\left(\frac{\delta_2^\Delta}{\mu_2} - 1\right) \psi\left(\frac{\delta_1^\Delta}{\mu_1} x_1\right)}{\left(1 - \frac{\delta_1^\Delta}{\mu_1}\right) x_1} \right| \\
&= \lim_{\Delta \rightarrow 0} \left| \frac{\frac{(1+\lambda_2)\delta_2^\Delta \log \delta_2}{\mu_2^2} \psi\left(\frac{\delta_1^\Delta}{\mu_1} x_1\right) + \left(\frac{\delta_2^\Delta}{\mu_2} - 1\right) \frac{d\psi\left(\frac{\delta_1^\Delta}{\mu_1} x_1\right)}{d\Delta}}{-\frac{(1+\lambda_1)\delta_1^\Delta \log \delta_1}{\mu_1^2} x_1 + \left(1 - \frac{\delta_1^\Delta}{\mu_1}\right) \frac{dx_1}{d\Delta}} \right| \\
&= \frac{(1 + \lambda_2) \log \delta_2}{(1 + \lambda_1) \log \delta_1} \times \frac{\psi(\bar{x}_1)}{\bar{x}_1} \\
&= \frac{\alpha}{1 - \alpha} \times \frac{\bar{x}_2}{\bar{x}_1}.
\end{aligned}$$

It follows that $\bar{x} = z^N$.

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