Announcements NAKE course "Game Theory", Fall semester 2011

Week 1, September 9, 2011

Subject material: Chapter 1, Chapter 2, Chapter 12 (Proof of Theorem 12.2 and Section 12.2 are optional).

Hand-in problems: 2.1(d), 2.2, 2.3, 12.1, 12.3, plus the extra problem 1 below (to be handed in on Friday, September 16).

1. Consider the following matrix game:

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Determine the optimal (maximin and minimax) strategies and the value of this game.

Week 2, September 16, 2011

Subject material: Chapter 3, Sections 13.1, 13.2 (13.2.3: optional), 13.3.

Hand-in problems: 3.6, 3.7, 13.5, 13.6, 13.11 (to be handed in on Friday, September 23).

Week 3, September 23, 2011

Subject material: Chapter 13, Sections 13.4, 13.7 (13.5 and 13.6 are optional).

Hand-in problems: 13.13, 13.17, 13.20 (to be handed in on Friday, September 30).

Week 4, September 30, 2011

Subject material: Chapters 4, 5, 14.

Hand-in problems: 4.8e,f, 14.9 (to be handed in on Friday, October 7).

Week 5, October 7, 2011

Subject material: Chapters 8.1, 15.1, 15.2

Hand-in problems: 15.1, 15.2, 15.3 (to be handed in on Friday, October 28).

Week 6, October 28, 2011

Subject material: Chapters 8.2, 15.3

Hand-in problems: 15.7, 15.11, and the following extra problem 2 (to be handed in on Friday, November 4).

2. Consider the following symmetric game.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}.$$

(a) Compute the set NE(A).

(b) Compute the set ST(A).

(c) Try to examine for each stationary strategy the behavior of the replicator dynamics close to that strategy.

NAKE Exam, Game Theory, 16-12-2009

This exam consists of 5 questions. Each exercise comprises 25 points. Your exam grade will be the the total number of points scored for all 5 questions divided by 10, with the only proviso that your grade cannot exceed 10.0.

Exercise 1. (Matrix games)

Consider the following matrix game:

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Determine the optimal (maximin and minimax) strategies and the value of this game.

Exercise 2. (Bimatrix games)

Consider the bimatrix game

$$(A, B) = \begin{pmatrix} 1,1 & 2,0 & 4,1 \\ 2,0 & 2,1 & 2,1 \end{pmatrix}.$$

Compute the best reply correspondences and make a picture. Determine all Nash equilibria of this game. Which one(s) is (are) perfect?

Exercise 3. (TU games)
(a)

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	1	3	1	5	7	7	14

(a) Compute the nucleolus of this TU game.
(b) Compute the Shapley value of this TU-game.
(c) Give a graphical representation of the core of this game. Also indicate the position of the Shapley value and the nucleolus.

Exercise 4. (Evolutionary games)

Consider the symmetric 3×3 game with payoff matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

(a) Compute the set $NE(A)$ of symmetric Nash equilibria of this game
(b) Compute the set $ST(A)$ of stationary points of this game.
(c) Compute the set $ESS(A)$ of evolutionarily stable strategies of this game. Don't forget to motivate your answers.

Extra exercise. (Bargaining sets)

Consider the TU game with player set $N = \{1,2,3,4,5\}$ and characteristic function

$$v(S) = \begin{cases} 6 & \text{if } S = N \\ 4 & \text{if } S \in \{(134),(135),(145),(234),(235),(245)\} \\ 0 & \text{else.} \end{cases}$$

(a) Show that $C(v) \neq \phi$.
(b) Define $x := ((3,3,0,0,0)$. Show that $x \notin C(v)$.

Now let $y = (y_2, y_3, y_4)$ be an allocation with $y_2 + y_3 + y_4 = 4$, $y_2 > 3$, $y_3 > 0$ and $y_4 > 0$.

(c) Note that y is an objection of 2 against 1 at x. Show that $z = (z_1, z_3, z_4)$ with $z_1 = y_2$, $z_3 = y_3$, and $z_4 = y_4$ is a counterobjection of 1 against 2.
(d) Also note that y is an objection of 3 against 1 at x. Show that $z = (z_1, z_4, z_5)$ with $z_1 = y_2$, $z_4 = y_4$, and $z_5 = y_3$ is a counterobjection of 1 against 3.
(d) Also note that y is an objection of 2 against 5 at x. Show that $z = (z_1, z_4, z_5)$ with $z_1 = y_2$, $z_4 = y_4$, and $z_5 = y_3$ is a counterobjection of 5 against 2.
(f) Show that x is an element of the bargaining set $B(v)$.