NOT FOR PUBLICATION

Additional Tables to "Panel Unit Root Tests in the Presence of Cross-Sectional Dependencies: Comparison and Implications for Modelling"

Christian Gengenbach, Franz C. Palm, Jean-Pierre Urbain*

Department of Quantitative Economics,
Universiteit Maastricht, The Netherlands
First version: September 16, 2004
This version: March 6, 2008

Monte Carlo simulation setup

In this section we study the small sample performance of the tests proposed by Pesaran (2007), Moon and Perron (2004) and Bai and Ng (2004a) for various types of DGPs. Furthermore, we consider the robust OLS t-test t_{rob} and the FGLS t-test t_{GLS} described in Breitung and Das (2008) and the recursive mean adjusted FGLS test $t_{\rho fglsrma}$ and the recursive mean adjusted test for the average data proposed by Sul (2007). All considered DGPs with one exception have the following structure which corresponds to Bai and Ng's (2004a) framework:

$$\begin{aligned} Y_{i,t} &= \lambda_i' F_t + E_{i,t}, \\ F_{m,t} &= \varphi F_{m,t-1} + f_{m,t}, \\ E_{i,t} &= \delta_i E_{i,t-1} + e_{i,t}, \end{aligned} \tag{1}$$

with $i = 1, \ldots, N$, $t = 1, \ldots, T$ and $m = 1, \ldots, K$. We consider three different values for N and T each, namely 20, 50 and 100[1]. The method of principle components estimates the space spanned by the common factors when N is large. We have chosen N and T at least equal to 20 to assure that common factors are estimated with sufficient precision or approximated

*Universiteit Maastricht, Faculty of Economics and Business Administration, P.O. Box 616, 6200 MD Maastricht, The Netherlands. Corresponding author: Christian Gengenbach, email: C.Gengenbach@ke.unimaas.nl. The authors would like to thank Jörg Breitung and four anonymous referees for helpful comments and suggestions. The usual disclaimer applies.

[1]Pesaran (2007) reports Monte Carlo results for his tests with $N, T = 10, 20, 30, 50, 100$, Moon and Perron (2004) choose $N = 10, 20$ and $T = 100, 300$, Bai and Ng (2004a) report results for $N = 40$, $T = 100$ while Bai and Ng (2007) choose $N, T = 20, 50, 100$, Sul (2007) performs simulations with $N = 5, 10, 15, 20$ and $T = 50, 100, 200$, and Breitung and Das (2008) select $N = 10, 20, 50$ and $T = 20, 50, 100$.

reasonably well by cross-sectional averages. Notice that the regularity condition $N \neq T$ needed for some tests is not satisfied in some cases. First a single common factor is considered, which is generated by a first order autoregression, or a random walk when $\varphi = 1$. We also consider the case of two common factors which are generated using the same parameter values for φ and σ_f^2, but different drawings for the error terms. The idiosyncratic terms $E_{i,t}$ are also generated by a first order autoregression or random walk with first order moving average, depending on whether or not $\delta_i = 1$.

In addition, a DGP as assumed by Pesaran (2007) and Moon and Perron (2004) is used:

$$\begin{aligned} Y_{i,t} &= \delta_i Y_{i,t-1} + u_{i,t}, \\ u_{i,t} &= \lambda_i f_t + e_{i,t}. \end{aligned} \tag{2}$$

In (1) and (2) the error terms are generated as MA(1) processes such that

$$\begin{aligned} f_{m,t} &= \eta_{m,t} + \gamma_m \eta_{m,t-1}, \\ e_{i,t} &= \varepsilon_{i,t} + \rho_i \varepsilon_{i,t-1}. \end{aligned}$$

The shocks are drawn from independent normal distributions, such that $\eta_t \sim i.i.d.N(0, \Sigma_f^2)$, with $\Sigma_f^2 = \sigma_f^2 I_K$, and $\varepsilon_{i,t} \sim i.i.d.N(0,1)$. We consider three different values for the signal-to-noise ratio, such that $\sigma_f^2 = 0.5$, 1 and 2^2. The MA parameters γ_m and ρ_i are independently, uniformly distributed on $[0.2, 0.5]$. The factor loading λ_i are uniformly distributed on $[-1, 3]$[3].

Three different types of non-stationarity are considered as null hypothesis, as well as different settings for the stationary alternative hypothesis. In particular, we consider the following 5 cases, where 1 to 4 use the DGP given by (1) and 5 uses DGP (2)[4]:

1. Common and idiosyncratic unit roots

$$H_0^A : \varphi = 1, \text{ and } \delta_i = 1 \text{ for all } i.$$

2. Common unit root, nearly stationary idiosyncratic components

$$H_0^B : \varphi = 1, \text{ and } \delta_i \sim U[0.8, 1] \text{ for all } i,$$

3. Stationary common component, integrated idiosyncratic components

$$H_0^C : \varphi = 0.95, \text{ and } \delta_i = 1 \text{ for all } i,$$

[2]In the tables we only report the values for $\sigma_f^2 = 1$. The other results are available at http://www.personeel.unimaas.nl/J.Urbain/.

[3]Consistency of the test procedure of Pesaran (2007) requires a non-zero mean for the factor loadings. This assumption is not necessary for the other approaches.

[4]Please note that under setup 1 (1) and (2) are equivalent. In cases 2, 4 and 5 we have stationarity provided $\delta_i \neq 0$.

4. Stationary common and idiosyncratic components

$$H_A^A : \varphi = 0.95 \text{ and } \delta_i \sim U[0.8, 1].$$

5. Stationary data using a DGP as given by (2) with heterogenous roots

$$H_A^E : \delta_i \sim U[0.8, 1] \text{ for all } i.$$

The results are obtained with GAUSS 8.0 using 1000 replications. The reported rejection frequencies are based on 5% nominal size. All power results are size unadjusted. For Pesaran's (2007) $CADF$ and $CIPS$ we use the critical values reported in Tables 1b and 3b of his paper. Results for Moon and Perron's (2004) statistics, Bai and Ng's (2004a) $P_{\hat{E}}^c$ statistic and Breitung and Das (2008) t_{rob} and t_{gls} are based on a critical value from the standard normal distribution. Rejection frequencies of the $ADF_{\hat{E}}^c$ and $ADF_{\hat{F}}^c$ statistics are obtained using the critical values from DF distributions for the no intercept and intercept only cases, respectively. Critical values for the MQ_c^c and MQ_f^c are provided in Table 1 of Bai and Ng (2004a). For Sul's (2007) $t_{\rho fglsrma}$ test we use finite sample critical values reported in Table 5 of Sul (2007) and for the t_{crma} we use the asymptotic critical value of -1.88. When obtaining the t_{crma} statistic we use $Y_{1,t}$ as covariate and calculate the cross-sectional averages over the remaining $N-1$ panel members such that $\bar{Y}_t = (N-1)^{-1} \sum_{i=2}^{N} Y_{i,t}$.

Similar to Moon and Perron (2004), we use the Andrews-Monahan (1992) estimator employing the quadratic spectral kernel in the estimation of the nuisance parameters for the t_a^* and t_b^* statistics. For Bai and Ng's (2004a) $ADF_{\hat{E}}^c$ and $ADF_{\hat{F}}^c$ and Pesaran's (2007) $CADF$ and $CIPS$ we use the Akaike information criterion (AIC) to determine the lag length, starting with a maximum lag length of $p_{max} = 6$. For the test of Sul (2007) and Breitung and Das (2008) we use the Bayesian information criterion (BIC). For the MQ_c^c statistic we use the Bartlett kernel with a bandwidth as suggested in Andrews (1991). The lag length for the MQ_f^c statistic is determined using the criteria proposed by Aznar and Salvador (2002).

Although the considered DGPs do not include deterministic components, we do account for individual fixed effects in the simulation by including constants in the regressions. Following the advise of Breitung and Das (2008) for the t_{rob} and t_{gls} test we consider data in deviation from the initial observation to remove the effect of an individual specific constant[5].

The finite sample performance of the considered test statistics depend on these choices. For reasons of comparison, we follow the original authors with the choices they report or we select a procedure that performs better in terms of size in our simulations.

[5]As already noted by Breitung and Das (2008), applying the tests to demeaned data leads to dependence on nuisance parameter unless applied to the GLS transformed data, and severe finite sample size distortions.

Table 1: Finite sample (average) rejection rates for DGP (1) with a single I(1) common factor and I(1) idiosyncratic components.

$\frac{\sigma_{\mathbf{f}}^{\mathbf{2}}}{\sigma_{\mathbf{e}}^{\mathbf{2}}}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.28	0.53	0.06	0.03	0.16	0.64	0.38	0.02	-	-	-
0.5	20	50	0.12	0.16	0.07	0.04	0.07	0.15	0.14	0.08	0.08	0.06	0.12
0.5	20	100	0.07	0.07	0.08	0.05	0.06	0.12	0.08	0.09	0.05	0.04	0.08
0.5	50	20	0.28	0.59	0.06	0.03	0.16	0.86	0.37	0.00	-	-	-
0.5	50	50	0.12	0.16	0.04	0.03	0.07	0.21	0.14	0.03	-	-	-
0.5	50	100	0.07	0.07	0.05	0.04	0.06	0.15	0.08	0.07	0.01	0.01	0.07
0.5	100	20	0.28	0.64	0.03	0.02	0.16	0.96	0.35	0.00	-	-	-
0.5	100	50	0.12	0.16	0.04	0.03	0.07	0.29	0.13	0.00	-	-	-
0.5	100	100	0.07	0.05	0.05	0.03	0.06	0.17	0.09	0.03	-	-	-
1	20	20	0.28	0.52	0.09	0.04	0.16	0.64	0.37	0.03	-	-	-
1	20	50	0.12	0.16	0.08	0.05	0.07	0.15	0.15	0.09	0.09	0.07	0.12
1	20	100	0.07	0.07	0.09	0.05	0.06	0.12	0.08	0.10	0.06	0.05	0.07
1	50	20	0.28	0.59	0.11	0.06	0.16	0.86	0.37	0.00	-	-	-
1	50	50	0.12	0.16	0.06	0.04	0.07	0.20	0.14	0.04	-	-	-
1	50	100	0.07	0.07	0.06	0.04	0.06	0.14	0.07	0.08	0.01	0.01	0.08
1	100	20	0.28	0.64	0.09	0.05	0.16	0.96	0.35	0.00	-	-	-
1	100	50	0.12	0.16	0.05	0.04	0.07	0.28	0.13	0.00	-	-	-
1	100	100	0.07	0.05	0.05	0.04	0.06	0.18	0.09	0.02	-	-	-
2	20	20	0.28	0.53	0.14	0.08	0.16	0.64	0.38	0.04	-	-	-
2	20	50	0.12	0.15	0.09	0.05	0.07	0.15	0.14	0.10	0.10	0.09	0.11
2	20	100	0.07	0.08	0.09	0.06	0.06	0.12	0.08	0.11	0.08	0.06	0.07
2	50	20	0.28	0.59	0.22	0.15	0.16	0.86	0.38	0.00	-	-	-
2	50	50	0.12	0.15	0.09	0.06	0.07	0.21	0.15	0.04	-	-	-
2	50	100	0.07	0.07	0.08	0.05	0.06	0.14	0.07	0.08	0.02	0.01	0.08
2	100	20	0.28	0.64	0.21	0.17	0.16	0.96	0.35	0.00	-	-	-
2	100	50	0.12	0.16	0.08	0.06	0.07	0.28	0.13	0.00	-	-	-
2	100	100	0.07	0.05	0.06	0.04	0.06	0.18	0.09	0.02	-	-	-

Finite sample (average) rejection rates for Pesaran's (2007) CADF and CIPS statistics, Moon and Perron's (2004) t_a^* and t_b^* statistics, Bai and Ng's (2004a) $ADF_{\hat{E}}^c$, $P_{\hat{E}}^c$, and $ADF_{\hat{F}}^c$ statistics, Breitung and Das's (2008) t_{rob} and t_{gls} statistics, and Sul's (2007) $t_{fglsrma}$ and t_{crma} statistics. Rejection frequencies are based on 5% cutoff values from Pesaran (2007), Tables 1b and 3b, Sul (2007) Table 5, 5% cutoff values of the standard normal distribution, or 5% Dickey-Fuller critical values for the test statistics as specified in the text. Results are obtained with GAUSS 8.0 using 1000 replications.

Table 2: Finite sample (average) rejection rates for DGP (1) with a single I(1) common factor and I(0) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.32	0.68	0.53	0.59	0.23	0.94	0.41	0.04	-	-	-
0.5	20	50	0.17	0.54	0.84	0.75	0.25	1.00	0.15	0.23	0.44	0.38	0.10
0.5	20	100	0.18	0.85	0.91	0.85	0.44	1.00	0.07	0.20	0.59	0.66	0.08
0.5	50	20	0.31	0.78	0.74	0.66	0.23	1.00	0.40	0.02	-	-	-
0.5	50	50	0.18	0.71	0.98	0.96	0.26	1.00	0.13	0.15	-	-	-
0.5	50	100	0.19	0.98	1.00	0.99	0.46	1.00	0.09	0.24	0.57	0.58	0.08
0.5	100	20	0.32	0.86	0.89	0.86	0.24	1.00	0.38	0.00	-	-	-
0.5	100	50	0.19	0.80	1.00	1.00	0.27	1.00	0.14	0.11	-	-	-
0.5	100	100	0.21	1.00	1.00	1.00	0.48	1.00	0.09	0.20	-	-	-
1	20	20	0.32	0.68	0.57	0.40	0.23	0.94	0.40	0.06	-	-	-
1	20	50	0.17	0.55	0.85	0.76	0.25	1.00	0.14	0.19	0.45	0.39	0.10
1	20	100	0.18	0.85	0.92	0.85	0.44	1.00	0.07	0.16	0.62	0.65	0.09
1	50	20	0.31	0.79	0.75	0.67	0.23	1.00	0.38	0.02	-	-	-
1	50	50	0.18	0.71	0.98	0.96	0.26	1.00	0.13	0.11	-	-	-
1	50	100	0.19	0.98	1.00	0.99	0.46	1.00	0.09	0.17	0.56	0.59	0.07
1	100	20	0.32	0.86	0.99	0.85	0.24	1.00	0.38	0.02	-	-	-
1	100	50	0.19	0.81	1.00	1.00	0.27	1.00	0.14	0.07	-	-	-
1	100	100	0.21	1.00	1.00	1.00	0.48	1.00	0.09	0.11	-	-	-
2	20	20	0.32	0.69	0.59	0.44	0.23	0.94	0.40	0.07	-	-	-
2	20	50	0.17	0.55	0.85	0.77	0.25	1.00	0.15	0.15	0.45	0.40	0.10
2	20	100	0.18	0.86	0.92	0.86	0.44	1.00	0.08	0.14	0.64	0.65	0.09
2	50	20	0.31	0.78	0.75	0.67	0.24	1.00	0.37	0.02	-	-	-
2	50	50	0.18	0.71	0.98	0.96	0.26	1.00	0.13	0.08	-	-	-
2	50	100	0.19	0.98	1.00	0.99	0.46	1.00	0.10	0.13	0.58	0.58	0.06
2	100	20	0.32	0.86	0.88	0.85	0.24	1.00	0.37	0.04	-	-	-
2	100	50	0.19	0.81	1.00	1.00	0.27	1.00	0.13	0.05	-	-	-
2	100	100	0.21	1.00	1.00	1.00	0.48	1.00	0.10	0.07	-	-	-

See notes Table 1.

Table 3: Finite sample (average) rejection rates for DGP (1) with a single I(0) common factor and I(1) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.27	0.47	0.08	0.04	0.16	0.62	0.40	0.02	-	-	-
0.5	20	50	0.11	0.10	0.09	0.05	0.07	0.12	0.18	0.18	0.09	0.08	0.19
0.5	20	100	0.06	0.02	0.10	0.05	0.06	0.12	0.19	0.34	0.07	0.06	0.22
0.5	50	20	0.27	0.55	0.07	0.04	0.16	0.84	0.40	0.00	-	-	-
0.5	50	50	0.11	0.08	0.06	0.04	0.07	0.20	0.17	0.07	-	-	-
0.5	50	100	0.05	0.01	0.06	0.04	0.06	0.12	0.19	0.25	0.00	0.01	0.24
0.5	100	20	0.27	0.59	0.04	0.03	0.16	0.95	0.41	0.00	-	-	-
0.5	100	50	0.11	0.06	0.05	0.04	0.07	0.25	0.16	0.00	-	-	-
0.5	100	100	0.06	0.01	0.05	0.04	0.06	0.15	0.18	0.14	-	-	-
1	20	20	0.27	0.47	0.13	0.06	0.16	0.61	0.41	0.04	-	-	-
1	20	50	0.11	0.10	0.11	0.06	0.07	0.12	0.19	0.22	0.11	0.10	0.18
1	20	100	0.06	0.02	0.12	0.06	0.06	0.12	0.19	0.43	0.09	0.09	0.20
1	50	20	0.27	0.55	0.17	0.10	0.16	0.84	0.40	0.00	-	-	-
1	50	50	0.11	0.08	0.09	0.06	0.07	0.20	0.17	0.08	-	-	-
1	50	100	0.05	0.01	0.08	0.06	0.06	0.12	0.19	0.31	0.01	0.01	0.23
1	100	20	0.27	0.58	0.13	0.08	0.16	0.95	0.40	0.00	-	-	-
1	100	50	0.11	0.06	0.08	0.06	0.07	0.25	0.16	0.01	-	-	-
1	100	100	0.06	0.01	0.07	0.06	0.06	0.15	0.19	0.09	-	-	-
2	20	20	0.27	0.46	0.21	0.10	0.16	0.61	0.41	0.07	-	-	-
2	20	50	0.12	0.10	0.16	0.09	0.07	0.12	0.18	0.25	0.13	0.12	0.15
2	20	100	0.06	0.02	0.17	0.09	0.06	0.12	0.18	0.47	0.13	0.12	0.18
2	50	20	0.27	0.55	0.31	0.23	0.16	0.84	0.40	0.01	-	-	-
2	50	50	0.11	0.08	0.18	0.12	0.07	0.19	0.16	0.10	-	-	-
2	50	100	0.05	0.01	0.16	0.11	0.06	0.12	0.20	0.38	0.02	0.02	0.21
2	100	20	0.27	0.57	0.31	0.25	0.16	0.95	0.40	0.00	-	-	-
2	100	50	0.11	0.06	0.18	0.13	0.07	0.25	0.16	0.03	-	-	-
2	100	100	0.06	0.00	0.13	0.10	0.06	0.14	0.19	0.13	-	-	-

See notes Table 1.

Table 4: Finite sample (average) rejection rates for DGP (1) with a single I(0) common factor and I(0) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.31	0.65	0.61	0.46	0.23	0.94	0.44	0.06	-	-	-
0.5	20	50	0.16	0.48	0.89	0.81	0.26	1.00	0.18	0.44	0.55	0.51	0.19
0.5	20	100	0.16	0.75	0.94	0.89	0.48	1.00	0.17	0.72	0.79	0.84	0.27
0.5	50	20	0.30	0.74	0.84	0.75	0.24	1.00	0.41	0.02	-	-	-
0.5	50	50	0.17	0.58	0.99	0.98	0.27	1.00	0.17	0.31	-	-	-
0.5	50	100	0.17	0.95	1.00	1.00	0.49	1.00	0.22	0.74	0.69	0.74	0.16
0.5	100	20	0.31	0.82	0.96	1.93	0.24	1.00	0.41	0.02	-	-	-
0.5	100	50	0.17	0.69	1.00	1.00	0.29	1.00	0.19	0.27	-	-	-
0.5	100	100	0.19	1.00	1.00	1.00	0.52	1.00	0.16	0.71	-	-	-
1	20	20	0.31	0.64	0.66	0.49	0.23	0.94	0.43	0.08	-	-	-
1	20	50	0.16	0.48	0.91	0.83	0.26	1.00	0.17	0.36	0.56	0.51	0.18
1	20	100	0.16	0.74	0.95	0.90	0.48	1.00	0.17	0.63	0.81	0.83	0.24
1	50	20	0.30	0.73	0.86	0.78	0.24	1.00	0.41	0.03	-	-	-
1	50	50	0.17	0.59	0.99	0.98	0.27	1.00	0.17	0.25	-	-	-
1	50	100	0.17	0.95	1.00	1.00	0.49	1.00	0.20	0.62	0.69	0.75	0.13
1	100	20	0.31	0.82	0.95	094	0.24	1.00	0.41	0.04	-	-	-
1	100	50	0.17	0.69	1.00	1.00	0.29	1.00	0.18	0.20	-	-	-
1	100	100	0.19	1.00	1.00	1.00	0.52	1.00	0.15	0.51	-	-	-
2	20	20	0.31	0.65	0.69	0.55	0.23	0.94	0.43	0.10	-	-	-
2	20	50	0.16	0.47	0.92	0.85	0.26	1.00	0.18	0.32	0.58	0.55	0.16
2	20	100	0.16	0.75	0.96	0.92	0.48	1.00	0.17	0.56	0.84	0.87	0.20
2	50	20	0.30	0.73	0.87	0.80	0.24	1.00	0.40	0.04	-	-	-
2	50	50	0.17	0.60	0.99	0.99	0.27	1.00	0.17	0.18	-	-	-
2	50	100	0.17	0.95	1.00	1.00	0.49	1.00	0.20	0.55	0.70	0.77	0.10
2	100	20	0.31	0.82	0.95	0.94	0.24	1.00	0.42	0.06	-	-	-
2	100	50	0.17	0.69	1.00	1.00	0.29	1.00	0.18	0.14	-	-	-
2	100	100	0.19	1.00	1.00	1.00	0.52	1.00	0.16	0.36	-	-	-

See notes Table 1.

Table 5: Finite sample (average) rejection rates for DGP (2) with a single I(0) common factor I(0) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$ADF^c_{\hat{E}}$	$P^c_{\hat{E}}$	$ADF^c_{\hat{F}}$	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.30	0.63	0.49	0.35	0.22	0.91	0.46	0.06	-	-	-
0.5	20	50	0.16	0.38	0.66	0.57	0.24	0.99	0.25	0.44	0.43	0.35	0.33
0.5	20	100	0.17	0.77	0.66	0.60	0.42	1.00	0.30	0.60	0.55	0.51	0.53
0.5	50	20	0.29	0.70	0.64	0.56	0.23	0.99	0.42	0.02	-	-	-
0.5	50	50	0.16	0.54	0.78	0.74	0.24	1.00	0.21	0.26	-	-	-
0.5	50	100	0.17	0.97	0.79	0.77	0.43	1.00	0.29	0.48	0.43	0.38	0.74
0.5	100	20	0.30	0.77	0.85	0.81	0.23	1.00	0.43	0.02	-	-	-
0.5	100	50	0.16	0.59	0.94	0.93	0.25	1.00	0.26	0.26	-	-	-
0.5	100	100	0.19	1.00	0.93	0.91	0.45	1.00	0.35	0.53	-	-	-
1	20	20	0.30	0.60	0.49	0.35	0.22	0.90	0.45	0.09	-	-	-
1	20	50	0.16	0.39	0.59	0.52	0.22	0.97	0.26	0.37	0.37	0.31	0.39
1	20	100	0.18	0.80	0.55	0.51	0.39	1.00	0.31	0.54	0.54	0.37	0.63
1	50	20	0.29	0.70	0.60	0.53	0.23	1.00	0.41	0.03	-	-	-
1	50	50	0.16	0.54	0.68	0.65	0.22	1.00	0.20	0.20	-	-	-
1	50	100	0.18	0.97	0.69	0.67	0.40	1.00	0.29	0.39	0.47	0.28	0.87
1	100	20	0.30	0.77	0.77	0.73	0.23	1.00	0.42	0.04	-	-	-
1	100	50	0.16	0.59	0.82	0.80	0.24	1.00	0.25	0.18	-	-	-
1	100	100	0.20	1.00	0.81	0.80	0.41	1.00	0.34	0.40	-	-	-
2	20	20	0.30	0.58	0.48	0.36	0.21	0.88	0.44	0.12	-	-	-
2	20	50	0.17	0.42	0.53	0.46	0.21	0.92	0.26	0.34	0.46	0.26	0.50
2	20	100	0.21	0.85	0.48	0.42	0.35	1.00	0.31	0.50	0.37	0.24	0.75
2	50	20	0.29	0.68	0.55	0.48	0.22	0.99	0.41	0.03	-	-	-
2	50	50	0.17	0.54	0.59	0.57	0.21	0.99	0.20	0.15	-	-	-
2	50	100	0.19	0.97	0.57	0.56	0.36	1.00	0.29	0.34	0.28	0.21	0.95
2	100	20	0.29	0.77	0.69	0.66	0.22	1.00	0.42	0.06	-	-	-
2	100	50	0.16	0.60	0.70	0.69	0.21	0.99	0.25	0.13	-	-	-
2	100	100	0.21	1.00	0.70	0.69	0.36	1.00	0.34	0.29	-	-	-

See notes Table 1.

Table 6: Finite sample (average) rejection rates for DGP (1) with two I(1) common factors and I(1) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.31	0.54	0.09	0.04	0.63	1.00	0.97	0.02	-	-	-
0.5	20	50	0.15	0.26	0.07	0.05	0.16	1.00	1.00	0.07	0.08	0.04	0.12
0.5	20	100	0.08	0.19	0.08	0.05	0.11	1.00	1.00	0.08	0.05	0.05	0.09
0.5	50	20	0.30	0.60	0.09	0.05	0.81	1.00	0.98	0.01	-	-	-
0.5	50	50	0.14	0.29	0.04	0.03	0.20	1.00	1.00	0.03	-	-	-
0.5	50	100	0.08	0.20	0.06	0.03	0.14	1.00	1.00	0.08	0.00	0.01	0.08
0.5	100	20	0.30	0.61	0.07	0.04	0.96	1.00	0.99	0.00	-	-	-
0.5	100	50	0.14	0.29	0.04	0.03	0.31	1.00	1.00	0.00	-	-	-
0.5	100	100	0.08	0.23	0.03	0.02	0.15	1.00	1.00	0.03	-	-	-
1	20	20	0.32	0.53	0.11	0.05	0.63	1.00	0.97	0.04	-	-	-
1	20	50	0.15	0.30	0.08	0.05	0.16	1.00	1.00	0.09	0.09	0.06	0.12
1	20	100	0.09	0.24	0.10	0.05	0.12	1.00	1.00	0.09	0.06	0.07	0.10
1	50	20	0.31	0.59	0.14	0.10	0.82	1.00	0.98	0.01	-	-	-
1	50	50	0.15	0.34	0.05	0.03	0.20	1.00	1.00	0.03	-	-	-
1	50	100	0.08	0.25	0.06	0.04	0.14	1.00	1.00	0.09	0.01	0.01	0.08
1	100	20	0.31	0.60	0.12	0.09	0.96	1.00	0.99	0.00	-	-	-
1	100	50	0.14	0.33	0.06	0.04	0.31	1.00	1.00	0.01	-	-	-
1	100	100	0.08	0.28	0.04	0.03	0.15	1.00	1.00	0.01	-	-	-
2	20	20	0.33	0.55	0.16	0.07	0.63	1.00	0.97	0.04	-	-	-
2	20	50	0.15	0.34	0.09	0.06	0.15	1.00	1.00	0.09	0.07	0.07	0.11
2	20	100	0.09	0.29	0.10	0.06	0.12	1.00	1.00	0.10	0.09	0.10	0.10
2	50	20	0.33	0.60	0.20	0.15	0.82	1.00	0.98	0.02	-	-	-
2	50	50	0.15	0.38	0.07	0.05	0.20	1.00	1.00	0.03	-	-	-
2	50	100	0.08	0.30	0.07	0.05	0.13	1.00	1.00	0.09	0.01	0.03	0.07
2	100	20	0.33	0.60	0.19	0.15	0.96	1.00	0.99	0.01	-	-	-
2	100	50	0.14	0.37	0.10	0.08	0.30	1.00	1.00	0.01	-	-	-
2	100	100	0.09	0.34	0.05	0.04	0.15	1.00	1.00	0.02	-	-	-

Finite sample (average) rejection rates for Pesaran's (2007) CADF and CIPS statistics, Moon and Perron's (2004) t_a^* and t_b^* statistics, Bai and Ng's (2004a) $P_{\hat{E}}^c$ statistic, Breitung and Das's (2008) t_{rob} and t_{gls} statistics, and Sul's (2007) $t_{fglsrma}$ and t_{crma} statistics. Proportions of repetitions when Bai and Ng's (2004a) MQ_c^c and MQ_f^c statistics chose the correct number of common stochastic trends. Rejection frequencies are based on 5% cutoff values from Pesaran (2007), Tables 1b and 3b, Sul (2007) Table 5, Bai and Ng (2004a) Table 1, 5% cutoff values of the standard normal distribution, or 5% Dickey-Fuller critical values for the test statistics as specified in the text. Results are obtained with GAUSS 8.0 using 1000 replications.

Table 7: Finite sample (average) rejection rates for DGP (1) with two I(1) common factors and I(0) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.32	0.60	0.47	0.36	0.90	1.00	0.97	0.04	-	-	-
0.5	20	50	0.17	0.45	0.83	0.75	0.99	1.00	1.00	0.15	0.41	0.33	0.11
0.5	20	100	0.14	0.54	0.92	0.87	1.00	1.00	1.00	0.16	0.59	0.61	0.10
0.5	50	20	0.33	0.67	0.71	0.61	1.00	1.00	0.98	0.03	-	-	-
0.5	50	50	0.17	0.54	0.98	0.96	1.00	1.00	1.00	0.10	-	-	-
0.5	50	100	0.14	0.60	1.00	0.99	1.00	1.00	1.00	0.14	0.48	0.54	0.07
0.5	100	20	0.33	0.71	0.86	0.82	1.00	1.00	0.98	0.03	-	-	-
0.5	100	50	0.17	0.54	1.00	1.00	1.00	1.00	1.00	0.06	-	-	-
0.5	100	100	0.13	0.61	1.00	1.00	1.00	1.00	1.00	0.12	-	-	-
1	20	20	0.33	0.59	0.48	0.37	0.91	1.00	0.98	0.05	-	-	-
1	20	50	0.16	0.44	0.84	0.75	0.99	1.00	1.00	0.13	0.42	0.35	0.11
1	20	100	0.13	0.48	0.93	0.88	1.00	1.00	1.00	0.12	0.64	0.62	0.09
1	50	20	0.33	0.63	0.72	0.62	1.00	1.00	0.98	0.04	-	-	-
1	50	50	0.17	0.51	0.98	0.96	1.00	1.00	1.00	0.06	-	-	-
1	50	100	0.12	0.55	1.00	0.99	1.00	1.00	1.00	0.11	0.47	0.56	0.06
1	100	20	0.34	0.68	0.85	0.81	1.00	1.00	0.98	0.03	-	-	-
1	100	50	0.16	0.51	1.00	1.00	1.00	1.00	1.00	0.04	-	-	-
1	100	100	0.12	0.54	1.00	1.00	1.00	1.00	1.00	0.07	-	-	-
2	20	20	0.34	0.57	0.49	0.37	0.90	1.00	0.98	0.05	-	-	-
2	20	50	0.16	0.44	0.84	0.76	0.99	1.00	1.00	0.11	0.45	0.38	0.11
2	20	100	0.12	0.45	0.93	0.88	1.00	1.00	1.00	0.10	0.65	0.62	0.09
2	50	20	0.34	0.61	0.71	0.63	1.00	1.00	0.98	0.04	-	-	-
2	50	50	0.17	0.50	0.98	0.96	1.00	1.00	1.00	0.05	-	-	-
2	50	100	0.12	0.50	1.00	0.99	1.00	1.00	1.00	0.10	0.50	0.60	0.05
2	100	20	0.35	0.65	0.84	0.81	1.00	1.00	0.99	0.03	-	-	-
2	100	50	0.16	0.50	1.00	1.00	1.00	1.00	1.00	0.04	-	-	-
2	100	100	0.11	0.48	1.00	1.00	1.00	1.00	1.00	0.06	-	-	-

See notes Table 6.

Table 8: Finite sample (average) rejection rates for DGP (1) with two I(0) common factors and I(1) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.31	0.53	0.12	0.05	0.61	0.00	0.00	0.03	-	-	-
0.5	20	50	0.14	0.20	0.09	0.06	0.13	0.00	0.00	0.25	0.10	0.08	0.21
0.5	20	100	0.08	0.10	0.11	0.07	0.10	0.00	0.00	0.48	0.08	0.09	0.28
0.5	50	20	0.30	0.58	0.12	0.08	0.78	0.00	0.00	0.01	-	-	-
0.5	50	50	0.13	0.19	0.06	0.05	0.15	0.00	0.00	0.11	-	-	-
0.5	50	100	0.07	0.06	0.08	0.05	0.11	0.00	0.00	0.45	0.01	0.01	0.27
0.5	100	20	0.30	0.60	0.11	0.07	0.94	0.00	0.00	0.00	-	-	-
0.5	100	50	0.13	0.23	0.07	0.05	0.21	0.00	0.00	0.01	-	-	-
0.5	100	100	0.08	0.09	0.05	0.03	0.13	0.00	0.00	0.20	-	-	-
1	20	20	0.32	0.54	0.16	0.08	0.59	0.00	0.00	0.06	-	-	-
1	20	50	0.14	0.24	0.13	0.08	0.14	0.00	0.00	0.30	0.10	0.11	0.21
1	20	100	0.08	0.15	0.15	0.09	0.10	0.00	0.00	0.57	0.11	0.14	0.29
1	50	20	0.31	0.59	0.23	0.16	0.78	0.00	0.00	0.02	-	-	-
1	50	50	0.14	0.26	0.12	0.08	0.14	0.00	0.00	0.13	-	-	-
1	50	100	0.07	0.12	0.14	0.10	0.11	0.00	0.00	0.53	0.01	0.04	0.24
1	100	20	0.31	0.59	0.25	0.19	0.94	0.00	0.00	0.00	-	-	-
1	100	50	0.14	0.31	0.15	0.11	0.21	0.00	0.00	0.06	-	-	-
1	100	100	0.08	0.18	0.12	0.09	0.12	0.00	0.00	0.28	-	-	-
2	20	20	0.34	0.57	0.24	0.14	0.59	0.00	0.00	0.07	-	-	-
2	20	50	0.15	0.30	0.19	0.11	0.14	0.00	0.00	0.33	0.13	0.16	0.21
2	20	100	0.09	0.21	0.22	0.13	0.10	0.00	0.00	0.64	0.17	0.21	0.30
2	50	20	0.33	0.60	0.33	0.26	0.78	0.00	0.00	0.03	-	-	-
2	50	50	0.15	0.33	0.26	0.18	0.14	0.00	0.00	0.13	-	-	-
2	50	100	0.08	0.22	0.30	0.20	0.11	0.00	0.00	0.57	0.03	0.09	0.20
2	100	20	0.33	0.60	0.37	0.31	0.94	0.00	0.00	0.02	-	-	-
2	100	50	0.14	0.37	0.33	0.26	0.21	0.00	0.00	0.11	-	-	-
2	100	100	0.08	0.27	0.30	0.24	0.12	0.00	0.00	0.36	-	-	-

See notes Table 6.

Table 9: Finite sample (average) rejection rates for DGP (1) with two I(0) common factors and I(0) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.32	0.59	0.59	0.45	0.89	0.00	0.00	0.08	-	-	-
0.5	20	50	0.17	0.46	0.89	0.85	1.00	0.00	0.00	0.50	0.63	0.54	0.21
0.5	20	100	0.15	0.64	0.96	0.92	1.00	0.00	0.00	0.82	0.86	0.87	0.29
0.5	50	20	0.33	0.69	0.86	0.78	1.00	0.00	0.00	0.05	-	-	-
0.5	50	50	0.18	0.56	1.00	0.99	1.00	0.00	0.00	0.38	-	-	-
0.5	50	100	0.16	0.73	1.00	1.00	1.00	0.00	0.00	0.85	0.70	0.82	0.11
0.5	100	20	0.33	0.72	0.96	0.95	1.00	0.00	0.00	0.06	-	-	-
0.5	100	50	0.17	0.55	1.00	1.00	1.00	0.00	0.00	0.34	-	-	-
0.5	100	100	0.14	0.73	1.00	1.00	1.00	0.00	0.00	0.76	-	-	-
1	20	20	0.33	0.59	0.62	0.48	0.90	0.00	0.00	0.10	-	-	-
1	20	50	0.17	0.47	0.91	0.86	1.00	0.00	0.00	0.43	0.67	0.60	0.22
1	20	100	0.14	0.62	0.97	0.93	1.00	0.00	0.00	0.77	0.89	0.88	0.29
1	50	20	0.34	0.67	0.87	0.80	1.00	0.00	0.00	0.06	-	-	-
1	50	50	0.18	0.53	1.00	0.99	1.00	0.00	0.00	0.32	-	-	-
1	50	100	0.14	0.68	1.00	1.00	1.00	0.00	0.00	0.77	0.70	0.88	0.07
1	100	20	0.35	0.69	0.96	0.94	1.00	0.00	0.00	0.07	-	-	-
1	100	50	0.17	0.54	1.00	1.00	1.00	0.00	0.00	0.27	-	-	-
1	100	100	0.15	0.67	1.00	1.00	1.00	0.00	0.00	0.66	-	-	-
2	20	20	0.35	0.58	0.66	0.51	0.89	0.00	0.00	0.11	-	-	-
2	20	50	0.17	0.47	0.92	0.87	1.00	0.00	0.00	0.40	0.66	0.64	0.21
2	20	100	0.14	0.59	0.98	0.94	1.00	0.00	0.00	0.74	0.92	0.90	0.30
2	50	20	0.35	0.65	0.88	0.82	1.00	0.00	0.00	0.07	-	-	-
2	50	50	0.18	0.53	1.00	0.99	1.00	0.00	0.00	0.25	-	-	-
2	50	100	0.14	0.63	1.00	1.00	1.00	0.00	0.00	0.73	0.75	0.92	0.07
2	100	20	0.35	0.65	0.96	0.94	1.00	0.00	0.00	0.08	-	-	-
2	100	50	0.17	0.53	1.00	1.00	1.00	0.00	0.00	0.25	-	-	-
2	100	100	0.13	0.63	1.00	1.00	1.00	0.00	0.00	0.59	-	-	-

See notes Table 6.

Table 10: Finite sample (average) rejection rates for DGP (2) with two I(0) common factors and I(0) idiosyncratic components.

$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.32	0.58	0.44	0.32	0.88	0.00	0.00	0.08	-	-	-
0.5	20	50	0.17	0.44	0.56	0.48	0.94	0.00	0.00	0.42	0.38	0.28	0.27
0.5	20	100	0.18	0.76	0.59	0.53	1.00	0.00	0.00	0.60	0.48	0.42	0.40
0.5	50	20	0.32	0.69	0.67	0.59	1.00	0.00	0.00	0.06	-	-	-
0.5	50	50	0.18	0.57	0.77	0.73	1.00	0.00	0.00	0.34	-	-	-
0.5	50	100	0.19	0.89	0.77	0.74	1.00	0.00	0.00	0.63	0.35	0.27	0.86
0.5	100	20	0.32	0.73	0.80	0.75	1.00	0.00	0.00	0.06	-	-	-
0.5	100	50	0.17	0.57	0.84	0.82	1.00	0.00	0.00	0.29	-	-	-
0.5	100	100	0.18	0.90	0.82	0.81	1.00	0.00	0.00	0.55	-	-	-
1	20	20	0.33	0.57	0.41	0.29	0.83	0.00	0.00	0.11	-	-	-
1	20	50	0.17	0.44	0.45	0.38	0.87	0.00	0.00	0.37	0.28	0.21	0.27
1	20	100	0.18	0.73	0.48	0.41	1.00	0.00	0.00	0.55	0.37	0.25	0.40
1	50	20	0.33	0.68	0.61	0.53	0.98	0.00	0.00	0.07	-	-	-
1	50	50	0.18	0.56	0.62	0.56	1.00	0.00	0.00	0.29	-	-	-
1	50	100	0.19	0.84	0.61	0.57	1.00	0.00	0.00	0.57	0.25	0.18	0.95
1	100	20	0.33	0.69	0.69	0.65	1.00	0.00	0.00	0.07	-	-	-
1	100	50	0.17	0.57	0.70	0.67	1.00	0.00	0.00	0.24	-	-	-
1	100	100	0.18	0.81	0.67	0.65	1.00	0.00	0.00	0.45	-	-	-
2	20	20	0.34	0.58	0.37	0.27	0.80	0.00	0.00	0.10	-	-	-
2	20	50	0.17	0.45	0.38	0.31	0.76	0.00	0.00	0.34	0.21	0.15	0.28
2	20	100	0.19	0.71	0.39	0.32	0.95	0.00	0.00	0.53	0.25	0.15	0.41
2	50	20	0.35	0.66	0.53	0.48	0.97	0.00	0.00	0.07	-	-	-
2	50	50	0.18	0.57	0.51	0.45	0.99	0.00	0.00	0.21	-	-	-
2	50	100	0.20	0.80	0.50	0.45	1.00	0.00	0.00	0.52	0.16	0.12	0.99
2	100	20	0.35	0.66	0.61	0.58	0.99	0.00	0.00	0.08	-	-	-
2	100	50	0.18	0.57	0.56	0.52	1.00	0.00	0.00	0.21	-	-	-
2	100	100	0.19	0.81	0.55	0.52	1.00	0.00	0.00	0.40	-	-	-

See notes Table 6.

Table 11: Finite sample (average) rejection rates for DGP (1) with two I(1) common factors and I(1) idiosyncratic components. The number of common factors is misspecified in the estimation.

			$\hat{\mathbf{K}} = \mathbf{1}$					$\hat{\mathbf{K}} = \mathbf{3}$				
$\frac{\sigma_f^2}{\sigma_e^2}$	**N**	**T**	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c
0.5	20	20	0.20	0.13	0.16	0.62	0.40	0.18	0.15	0.61	0.00	0.01
0.5	20	50	0.21	0.14	0.07	0.22	0.12	0.11	0.08	0.16	0.00	0.00
0.5	20	100	0.22	0.15	0.06	0.17	0.08	0.11	0.07	0.12	0.00	0.00
0.5	50	20	0.22	0.18	0.16	0.79	0.39	0.29	0.28	0.83	0.00	0.01
0.5	50	50	0.24	0.20	0.07	0.27	0.12	0.18	0.17	0.19	0.00	0.00
0.5	50	100	0.27	0.22	0.06	0.21	0.09	0.15	0.12	0.13	0.00	0.00
0.5	100	20	0.32	0.29	0.16	0.86	0.38	0.29	0.29	0.92	0.00	0.01
0.5	100	50	0.37	0.35	0.07	0.39	0.14	0.20	0.19	0.30	0.00	0.00
0.5	100	100	0.39	0.37	0.06	0.34	0.10	0.12	0.11	0.18	0.00	0.00
1	20	20	0.31	0.22	0.16	0.58	0.40	0.25	0.22	0.61	0.00	0.01
1	20	50	0.32	0.23	0.07	0.25	0.12	0.18	0.14	0.16	0.00	0.00
1	20	100	0.32	0.23	0.06	0.21	0.08	0.15	0.11	0.13	0.00	0.00
1	50	20	0.38	0.32	0.16	0.75	0.39	0.37	0.36	0.82	0.00	0.01
1	50	50	0.38	0.33	0.07	0.32	0.13	0.25	0.26	0.19	0.00	0.00
1	50	100	0.41	0.37	0.06	0.26	0.09	0.22	0.21	0.12	0.00	0.00
1	100	20	0.45	0.42	0.16	0.78	0.38	0.37	0.36	0.92	0.00	0.01
1	100	50	0.48	0.46	0.07	0.42	0.14	0.30	0.29	0.30	0.00	0.00
1	100	100	0.50	0.49	0.06	0.38	0.10	0.22	0.21	0.18	0.00	0.00
2	20	20	0.38	0.31	0.16	0.56	0.40	0.29	0.25	0.62	0.00	0.01
2	20	50	0.41	0.33	0.07	0.29	0.12	0.23	0.19	0.16	0.00	0.00
2	20	100	0.41	0.34	0.06	0.26	0.09	0.21	0.16	0.13	0.00	0.00
2	50	20	0.47	0.44	0.16	0.70	0.38	0.41	0.40	0.82	0.00	0.00
2	50	50	0.47	0.44	0.07	0.33	0.13	0.32	0.31	0.20	0.00	0.00
2	50	100	0.49	0.47	0.06	0.32	0.08	0.29	0.27	0.12	0.00	0.00
2	100	20	0.50	0.49	0.16	0.69	0.38	0.41	0.40	0.91	0.00	0.01
2	100	50	0.52	0.51	0.07	0.42	0.14	0.35	0.34	0.30	0.00	0.00
2	100	100	0.53	0.53	0.07	0.39	0.10	0.28	0.27	0.18	0.00	0.00

Finite sample (average) rejection rates for Moon and Perron's (2004) t_a^* and t_b^* statistics, and Bai and Ng's (2004a) $ADF_{\hat{E}}^c$, $P_{\hat{E}}^c$, and $ADF_{\hat{F}}^c$ statistics, the proportions of repetitions when Bai and Ng's (2004a) MQ_c^c and MQ_f^c statistics chose the correct number of common stochastic trends, when the number of common factors is misspecified. Rejection frequencies are based on Bai and Ng (2004a) Table 1, 5% cutoff values of the standard normal distribution, or 5% Dickey-Fuller critical values for the test statistics as specified in the text. Results are obtained with GAUSS 8.0 using 1000 replications.

Table 12: Finite sample (average) rejection rates for DGP (1) with two I(1) common factors and I(0) idiosyncratic components. The number of common factors is misspecified in the estimation.

			$\hat{\mathbf{K}} = \mathbf{1}$					$\hat{\mathbf{K}} = \mathbf{3}$				
$\frac{\sigma_{\mathbf{f}}^2}{\sigma_{\mathbf{e}}^2}$	**N**	**T**	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c
0.5	20	20	0.45	0.38	0.20	0.76	0.38	0.38	0.30	0.82	0.00	0.02
0.5	20	50	0.59	0.53	0.16	0.66	0.14	0.77	0.68	0.96	0.00	0.00
0.5	20	100	0.66	0.60	0.21	0.73	0.08	0.90	0.85	1.00	0.00	0.00
0.5	50	20	0.61	0.58	0.20	0.88	0.40	0.54	0.48	0.99	0.00	0.02
0.5	50	50	0.63	0.61	0.16	0.73	0.14	0.95	0.92	1.00	0.00	0.00
0.5	50	100	0.71	0.70	0.22	0.81	0.10	0.99	0.99	1.00	0.00	0.00
0.5	100	20	0.69	0.67	0.20	0.92	0.37	0.68	0.64	1.00	0.00	0.03
0.5	100	50	0.73	0.72	0.15	0.77	0.13	1.00	0.99	1.00	0.00	0.00
0.5	100	100	0.77	0.76	0.21	0.84	0.08	1.00	1.00	1.00	0.00	0.00
1	20	20	0.46	0.40	0.19	0.69	0.38	0.41	0.31	0.84	0.00	0.01
1	20	50	0.56	0.49	0.14	0.56	0.15	0.78	0.69	0.96	0.00	0.00
1	20	100	0.60	0.54	0.18	0.61	0.08	0.90	0.86	1.00	0.00	0.00
1	50	20	0.58	0.56	0.19	0.79	0.39	0.54	0.49	0.99	0.00	0.03
1	50	50	0.57	0.56	0.14	0.64	0.14	0.95	0.92	1.00	0.00	0.00
1	50	100	0.65	0.62	0.19	0.70	0.09	0.99	0.99	1.00	0.00	0.00
1	100	20	0.64	0.62	0.20	0.83	0.37	0.68	0.64	1.00	0.00	0.03
1	100	50	0.67	0.66	0.13	0.66	0.12	1.00	0.99	1.00	0.00	0.00
1	100	100	0.70	0.69	0.18	0.73	0.08	1.00	1.00	1.00	0.00	0.00
2	20	20	0.47	0.41	0.18	0.61	0.38	0.42	0.33	0.85	0.00	0.02
2	20	50	0.52	0.47	0.13	0.47	0.14	0.78	0.69	0.97	0.00	0.00
2	20	100	0.55	0.50	0.15	0.52	0.08	0.91	0.86	1.00	0.00	0.00
2	50	20	0.57	0.55	0.19	0.70	0.39	0.54	0.49	0.99	0.00	0.02
2	50	50	0.53	0.52	0.12	0.54	0.14	0.95	0.92	1.00	0.00	0.00
2	50	100	0.59	0.57	0.15	0.58	0.09	0.99	0.99	1.00	0.00	0.00
2	100	20	0.61	0.60	0.19	0.73	0.37	0.68	0.64	1.00	0.00	0.03
2	100	50	0.62	0.62	0.12	0.56	0.12	1.00	0.99	1.00	0.00	0.00
2	100	100	0.66	0.66	0.15	0.64	0.08	1.00	1.00	1.00	0.00	0.00

See notes Table 11.

Table 13: Finite sample (average) rejection rates for DGP (1) with two I(0) common factors and I(1) idiosyncratic components. The number of common factors is misspecified in the estimation.

			$\hat{\mathbf{K}} = \mathbf{1}$					$\hat{\mathbf{K}} = \mathbf{3}$				
$\frac{\sigma_{\mathbf{f}}^2}{\sigma_{\mathbf{e}}^2}$	**N**	**T**	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c
0.5	20	20	0.29	0.49	0.17	0.65	0.42	0.30	0.26	0.57	0.00	0.00
0.5	20	50	0.39	0.70	0.09	0.30	0.17	0.27	0.21	0.13	0.00	0.00
0.5	20	100	0.49	0.81	0.08	0.33	0.16	0.27	0.19	0.10	0.00	0.00
0.5	50	20	0.36	0.28	0.16	0.83	0.42	0.50	0.48	0.80	0.00	0.00
0.5	50	50	0.46	0.35	0.08	0.41	0.16	0.56	0.53	0.15	0.00	0.00
0.5	50	100	0.60	0.49	0.08	0.45	0.18	0.56	0.53	0.10	0.00	0.00
0.5	100	20	0.49	0.44	0.17	0.92	0.41	0.55	0.54	0.90	0.00	0.00
0.5	100	50	0.67	0.61	0.09	0.62	0.20	0.63	0.61	0.25	0.00	0.00
0.5	100	100	0.81	0.75	0.09	0.68	0.20	0.68	0.66	0.15	0.00	0.00
1	20	20	0.45	0.33	0.17	0.63	0.42	0.42	0.38	0.57	0.00	0.00
1	20	50	0.62	0.48	0.10	0.39	0.17	0.52	0.45	0.14	0.00	0.00
1	20	100	0.77	0.64	0.10	0.48	0.15	0.59	0.52	0.10	0.00	0.00
1	50	20	0.56	0.50	0.17	0.82	0.41	0.62	0.60	0.79	0.00	0.00
1	50	50	0.74	0.66	0.09	0.50	0.16	0.76	0.75	0.15	0.00	0.00
1	50	100	0.89	0.83	0.09	0.63	0.18	0.89	0.89	0.11	0.00	0.00
1	100	20	0.64	0.60	0.17	0.87	0.41	0.67	0.67	0.90	0.00	0.00
1	100	50	0.85	0.84	0.10	0.68	0.19	0.81	0.81	0.24	0.00	0.00
1	100	100	0.97	0.95	0.10	0.82	0.20	0.94	0.94	0.14	0.00	0.00
2	20	20	0.56	0.46	0.18	0.61	0.43	0.48	0.44	0.56	0.00	0.00
2	20	50	0.76	0.66	0.10	0.49	0.17	0.66	0.60	0.13	0.00	0.00
2	20	100	0.94	0.86	0.12	0.65	0.16	0.82	0.77	0.10	0.00	0.00
2	50	20	0.65	0.61	0.18	0.79	0.40	0.65	0.63	0.79	0.00	0.00
2	50	50	0.84	0.81	0.10	0.58	0.16	0.83	0.82	0.15	0.00	0.00
2	50	100	0.97	0.96	0.11	0.77	0.17	0.95	0.94	0.11	0.00	0.00
2	100	20	0.70	0.68	0.17	0.81	0.41	0.71	0.71	0.90	0.00	0.00
2	100	50	0.90	0.89	0.11	0.72	0.19	0.87	0.87	0.24	0.00	0.00
2	100	100	0.99	0.98	0.13	0.89	0.19	0.97	0.97	0.14	0.00	0.00

See notes Table 11.

Table 14: Finite sample (average) rejection rates for DGP (1) with two I(0) common factors and I(0) idiosyncratic components. The number of common factors is misspecified in the estimation.

			$\hat{\mathbf{K}} = \mathbf{1}$					$\hat{\mathbf{K}} = \mathbf{3}$				
$\frac{\sigma_{\mathbf{f}}^2}{\sigma_{\mathbf{e}}^2}$	**N**	**T**	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c
0.5	20	20	0.63	0.53	0.21	0.84	0.42	0.54	0.44	0.83	0.00	0.00
0.5	20	50	0.90	0.86	0.23	0.93	0.17	0.90	0.84	0.99	0.00	0.00
0.5	20	100	0.99	0.98	0.43	1.00	0.19	0.96	0.94	1.00	0.00	0.00
0.5	50	20	0.78	0.76	0.22	0.93	0.43	0.75	0.69	0.99	0.00	0.00
0.5	50	50	0.98	0.97	0.23	0.98	0.17	1.00	0.99	1.00	0.00	0.00
0.5	50	100	1.00	1.00	0.43	1.00	0.18	1.00	1.00	1.00	0.00	0.00
0.5	100	20	0.86	0.85	0.22	0.97	0.40	0.87	0.84	1.00	0.00	0.00
0.5	100	50	0.99	0.99	0.22	0.98	0.16	1.00	1.00	1.00	0.00	0.00
0.5	100	100	1.00	1.00	0.42	1.00	0.17	1.00	1.00	1.00	0.00	0.00
1	20	20	0.63	0.57	0.21	0.78	0.42	0.58	0.50	0.83	0.00	0.00
1	20	50	0.89	0.85	0.23	0.87	0.17	0.93	0.87	0.99	0.00	0.00
1	20	100	0.99	0.98	0.40	1.00	0.19	0.98	0.96	1.00	0.00	0.00
1	50	20	0.75	0.73	0.22	0.88	0.43	0.77	0.72	0.99	0.00	0.00
1	50	50	0.96	0.95	0.22	0.94	0.17	1.00	1.00	1.00	0.00	0.00
1	50	100	1.00	1.00	0.40	1.00	0.18	1.00	1.00	1.00	0.00	0.00
1	100	20	0.82	0.82	0.21	0.90	0.41	0.87	0.84	1.00	0.00	0.00
1	100	50	0.97	0.97	0.21	0.95	0.16	1.00	1.00	1.00	0.00	0.00
1	100	100	1.00	1.00	0.39	1.00	0.17	1.00	1.00	1.00	0.00	0.00
2	20	20	0.63	0.58	0.21	0.72	0.42	0.60	0.53	0.84	0.00	0.00
2	20	50	0.88	0.84	0.22	0.80	0.17	1.00	0.88	0.99	0.00	0.00
2	20	100	0.99	0.98	0.38	0.98	0.19	1.00	0.98	1.00	0.00	0.00
2	50	20	0.72	0.70	0.21	0.79	0.43	0.77	0.72	0.99	0.00	0.00
2	50	50	0.94	0.92	0.21	0.88	0.17	1.00	1.00	1.00	0.00	0.00
2	50	100	1.00	1.00	0.38	1.00	0.18	1.00	1.00	1.00	0.00	0.00
2	100	20	0.80	0.79	0.21	0.83	0.40	0.86	0.85	1.00	0.00	0.00
2	100	50	0.95	0.95	0.20	0.90	0.16	1.00	1.00	1.00	0.00	0.00
2	100	100	1.00	1.00	0.37	1.00	0.17	1.00	1.00	1.00	0.00	0.00

See notes Table 11.

Table 15: Finite sample (average) rejection rates for DGP (2) with two I(0) common factors and I(0) idiosyncratic components. The number of common factors is misspecified in the estimation.

			$\hat{\mathbf{K}} = \mathbf{1}$					$\hat{\mathbf{K}} = \mathbf{3}$				
$\frac{\sigma_{\mathbf{f}}^2}{\sigma_{\mathbf{e}}^2}$	**N**	**T**	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_a^*	t_b^*	$P_{\hat{E}}^c$	MQ_c^c	MQ_f^c
0.5	20	20	0.51	0.42	0.22	0.87	0.45	0.46	0.39	0.79	0.00	0.00
0.5	20	50	0.64	0.56	0.26	0.97	0.27	0.63	0.58	0.91	0.00	0.00
0.5	20	100	0.68	0.61	0.46	1.00	0.40	0.69	0.63	1.00	0.00	0.00
0.5	50	20	0.73	0.68	0.23	0.97	0.46	0.67	0.62	0.97	0.00	0.00
0.5	50	50	0.88	0.85	0.26	1.00	0.27	0.86	0.83	1.00	0.00	0.00
0.5	50	100	0.90	0.88	0.48	1.00	0.37	0.88	0.85	1.00	0.00	0.00
0.5	100	20	0.80	0.78	0.23	0.99	0.42	0.76	0.73	1.00	0.00	0.00
0.5	100	50	0.89	0.88	0.25	1.00	0.22	0.93	0.92	1.00	0.00	0.00
0.5	100	100	0.92	0.91	0.46	1.00	0.32	0.96	0.95	1.00	0.00	0.00
1	20	20	0.51	0.43	0.22	0.83	0.44	0.53	0.46	0.75	0.00	0.00
1	20	50	0.57	0.50	0.25	0.94	0.27	0.66	0.60	0.84	0.00	0.00
1	20	100	0.62	0.55	0.45	1.00	0.40	0.67	0.62	0.99	0.00	0.00
1	50	20	0.69	0.65	0.47	0.93	0.46	0.70	0.67	0.96	0.00	0.00
1	50	50	0.82	0.78	0.27	0.99	0.27	0.88	0.87	1.00	0.00	0.00
1	50	100	0.85	0.82	0.37	1.00	0.36	0.87	0.86	1.00	0.00	0.00
1	100	20	0.75	0.74	0.22	0.96	0.43	0.80	0.78	0.99	0.00	0.00
1	100	50	0.82	0.81	0.24	0.99	0.22	0.94	0.93	1.00	0.00	0.00
1	100	100	0.86	0.84	0.47	1.00	0.31	0.94	0.93	1.00	0.00	0.00
2	20	20	0.50	0.42	0.21	0.77	0.44	0.60	0.54	0.71	0.00	0.00
2	20	50	0.54	0.48	0.24	0.90	0.27	0.67	0.61	0.77	0.00	0.00
2	20	100	0.58	0.51	0.44	1.00	0.39	0.62	0.57	0.97	0.00	0.00
2	50	20	0.67	0.64	0.22	0.87	0.46	0.77	0.74	0.95	0.00	0.00
2	50	50	0.75	0.73	0.24	0.97	0.27	0.87	0.87	0.99	0.00	0.00
2	50	100	0.80	0.76	0.45	1.00	0.35	0.84	0.81	1.00	0.00	0.00
2	100	20	0.72	0.71	0.22	0.90	0.43	0.83	0.82	0.99	0.00	0.00
2	100	50	0.77	0.75	0.23	0.97	0.22	0.94	0.93	1.00	0.00	0.00
2	100	100	0.80	0.78	0.43	1.00	0.31	0.89	0.88	1.00	0.00	0.00

See notes Table 11.

Table 16: Finite sample (average) rejection rates for DGP (1) without MA serial correlation and with known lag length. A single I(1) common factor and I(1) idiosyncratic components is present in the DGP.

$\frac{\sigma_{\mathbf{f}}^{\mathbf{2}}}{\sigma_{\mathbf{e}}^{\mathbf{2}}}$	**N**	**T**	CADF	CIPS	t_a^*	t_b^*	$ADF_{\hat{E}}^c$	$P_{\hat{E}}^c$	$ADF_{\hat{F}}^c$	t_{rob}	t_{gls}	$t_{fglsrma}$	t_{crma}
0.5	20	20	0.10	0.22	0.10	0.06	0.06	0.10	0.09	0.04	-	-	-
0.5	20	50	0.06	0.10	0.10	0.06	0.05	0.06	0.05	0.08	0.02	0.02	0.06
0.5	20	100	0.06	0.08	0.11	0.07	0.05	0.06	0.06	0.08	0.04	0.03	0.06
0.5	50	20	0.10	0.26	0.07	0.05	0.06	0.11	0.10	0.03	-	-	-
0.5	50	50	0.07	0.13	0.08	0.05	0.05	0.08	0.08	0.07	-	-	-
0.5	50	100	0.06	0.07	0.07	0.05	0.05	0.06	0.06	0.06	0.00	0.00	0.06
0.5	100	20	0.10	0.33	0.05	0.05	0.06	0.12	0.09	0.03	-	-	-
0.5	100	50	0.07	0.15	0.06	0.06	0.05	0.09	0.06	0.06	-	-	-
0.5	100	100	0.06	0.10	0.06	0.06	0.05	0.07	0.07	0.07	-	-	-
1	20	20	0.10	0.22	0.10	0.05	0.06	0.10	0.10	0.06	-	-	-
1	20	50	0.06	0.10	0.10	0.06	0.05	0.08	0.07	0.08	0.01	0.02	0.05
1	20	100	0.06	0.08	0.11	0.07	0.05	0.08	0.06	0.08	0.04	0.03	0.05
1	50	20	0.10	0.25	0.06	0.04	0.06	0.09	0.08	0.05	-	-	-
1	50	50	0.07	0.12	0.07	0.05	0.05	0.07	0.07	0.09	-	-	-
1	50	100	0.06	0.07	0.07	0.05	0.05	0.06	0.06	0.10	0.00	0.00	0.05
1	100	20	0.10	0.33	0.06	0.04	0.06	0.10	0.09	0.06	-	-	-
1	100	50	0.07	0.15	0.07	0.05	0.05	0.08	0.06	0.09	-	-	-
1	100	100	0.06	0.10	0.07	0.05	0.05	0.08	0.04	0.08	-	-	-
2	20	20	0.10	0.22	0.09	0.05	0.06	0.09	0.10	0.08	-	-	-
2	20	50	0.07	0.10	0.10	0.05	0.05	0.08	0.08	0.08	0.03	0.03	0.06
2	20	100	0.06	0.08	0.10	0.07	0.05	0.07	0.06	0.10	0.04	0.05	0.06
2	50	20	0.09	0.26	0.05	0.03	0.06	0.09	0.10	0.06	-	-	-
2	50	50	0.07	0.12	0.07	0.05	0.05	0.07	0.07	0.09	-	-	-
2	50	100	0.06	0.07	0.07	0.04	0.05	0.06	0.06	0.09	0.01	0.00	0.04
2	100	20	0.09	0.33	0.07	0.05	0.06	0.11	0.09	0.07	-	-	-
2	100	50	0.07	0.15	0.07	0.05	0.05	0.10	0.07	0.09	-	-	-
2	100	100	0.06	0.10	0.08	0.05	0.05	0.06	0.07	0.08	-	-	-

Finite sample (average) rejection rates for Pesaran's (2007) CADF and CIPS statistics, Moon and Perron's (2004) t_a^* and t_b^* statistics, Bai and Ng's (2004a) $ADF_{\hat{E}}^c$, $P_{\hat{E}}^c$, and $ADF_{\hat{F}}^c$ statistics, Breitung and Das's (2008) t_{rob} and t_{gls} statistics, and Sul's (2007) $t_{fglsrma}$ and t_{crma} statistics. Rejection frequencies are based on 5% cutoff values from Pesaran (2007), Tables 1b and 3b, Sul (2007) Table 5, 5% cutoff values of the standard normal distribution, or 5% Dickey-Fuller critical values for the test statistics as specified in the text. Results are obtained with GAUSS 8.0 using 1000 replications.